the hydrodynamic theory of multiple production

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Simple estimates are presented for a number of characteristics of the indicated reaction within the framework of the hydrodynamic approach. The experimental data are in qualitative agreement with these estimates, and deviate in a number of respects from the conclusions of the parton model.

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Interest in the Landau hydrodynamic theory $^{[1]}$ has increased of late because the experimental data on hadron collisions turned out to agree with its predictions. $^{[2]}$ The present paper is devoted to an analysis of e^*e^- annihilation into hadrons and hadron-nuclear collisions from the point of view of this theory. We must emphasize that the analysis is qualitative, since neither our estimates nor the data under the discussion can claim high accuracy.

The main difference between the case of e^+e^- annihilation and hadron collisions is that the energy is released in a small space-time region and the hydrodynamic spreading is radial in character rather than quasi-one-dimensional. Experiments 141 have indeed demonstrated the isotropy of the secondary particles, and the ordered picture predicted by the parton model was not observed.

Simple thermodynamic relations^[3] lead to the following expression for the average secondary-particle energy $\langle E \rangle$:

$$< E > \alpha (E_{tot} / V_o)^{\frac{c^2}{1+c^2}}$$

where $E_{\rm tot}$ is the total energy; we have in mind an equation of state $p=c^2\epsilon$ (p is the pressure and ϵ the energy density), with the constant parameter c^2 having the meaning of the square of the speed of sound. V_0 is the initial volume, and its estimate calls for additional assumptions. The quantity c^2 was estimated in $^{[5]}$ by two methods: by direct calculations by the Beth-Uhlenbeck method, and from data on the multiplicity distribution. Both methods yield $c^2\approx 0.2$. The data of $c^{[4]}$ on the

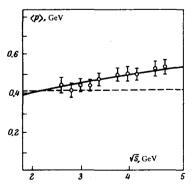


FIG. 1. Average momentum of secondary particles in e^+e^- annihilation in hadrons as a function of the total energy. ^[4] The dashed line corresponds to a thermal spectrum with $T = m_{\pi}$, and the solid line is a plot of $\langle p \rangle = 0.34 s^{1/8}$.

average momentum of the secondary particles are shown in Fig. 1. Most secondary particles are ultrarelativistic pions, so that $\langle \, p \rangle \approx \langle E \rangle$. The observed dependence of $\langle \, p \rangle$ on the total energy agrees with (1) at $V_0 \approx {\rm const}$, and contradicts the scaling relation $\langle E \rangle \sim \alpha \, E_{\rm tot}$ predicted by the parton model.

The calculation of the secondary-particle energy distribution calls for the solution of the equations of relativistic hydrodynamics. $^{\{3,6\}}$ At sufficiently low energies $E_{\rm tot} \lesssim 10$ GeV, however, it is important only that the rapidity distribution of the collective motion have a peak at a certain value y_0 that depends on the energy. Therefore the hydrodynamic effects reduce to a Doppler shift of the thermal spectrum. It retains its exponential character with a slope determined from (1). The data presented in Fig. 2 agree well with this result. This behavior of the spectra may be confused with an increase of temperature, but the difference becomes clear when the composition of the secondary particles, corresponding to a fixed final temperature in hydrodynamic theory, is discussed.

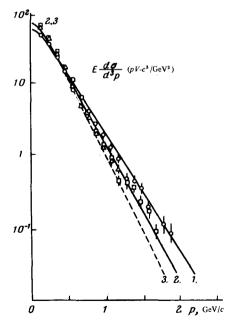


FIG. 2. Spectra of the secondary particles in e^+e^- annihilation into hadrons ^[4] at the following total energies \sqrt{s} : $\bigcirc -4.8$ GeV, $\square -3.8$ GeV, $\triangle -3.0$ GeV. The curves correspond to the hydrodynamic model at energies: 1-5 GeV, 2-3 GeV. The thermal spectrum with T=160 MeV is shown for comparison (dashed curve 3).

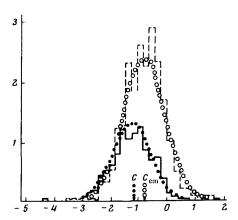


FIG. 3. Distribution with respect to log $\tan\theta$ for pp collisions (solid curve and points) and for collisions of photons with emulsion nuclei (dashed histogram and open circles). The histogram corresponds to the data of [11,9] at 200 GeV, and the points correspond to the model described in the text.

We now proceed to a discussion of the collisions of hadrons with nuclei. A relatively detailed hydrodynamic theory of this phenomenon was developed in [8]. It is based on an analysis of the production of entropy in shock waves, the accuracy of which is uncertain because the dissipative and quantum effect, which are significant at the initial stage, are not taken into account. Simple qualitative estimates can be obtained on the basis of the model assumption proposed by Gottfried (see also [10]). It consists in the fact that the incident hadron and the tube that it knocks out from the nucleus constitute a single statistical system during the initial stage. This case then reduces by simple calculation to hadron-hadron collision. We present the dependence of the average multiplicity on the atomic number A:

$$R_A = \frac{\langle N \rangle_{hA}}{\langle N \rangle_{hh}} = A^{\frac{1}{6} \frac{1-c^2}{1+c^2}},$$
 (2)

which was obtained by substituting in the usual formula for the multiplicity the invariant $s'\alpha sA^{1/3}$ of the hadrontube collision in place $s=2p_{\rm LAB}m$. At $c^2=0.2$, the exponent in (2) is equal to 0.11, as against 0.131 ± 0.005 in $^{[14]}$, and the agreement is reasonable. At the same time, in the parton model we obtain $^{[14]}$ $R_A=1+{\rm const}\cdot A^{1/3}/{\rm lns}$, which disagrees with the approximate constancy of R_A in a wide energy interval $10-10^4$ GeV. $^{[9]}$

The predicted rapidity distribution of the secondary particles is approximately Gaussian, with a center at the point y_c corresponding to the hadron-tube rest system: $y_c \approx (1/2) \ln(E_{\rm LAB}/2m) + \ln A/6$. The data of [111], which are shown in Fig. 3, agree well with these conclusions. We note that according to the parton model the curves for the nuclei and nucleons should coincide in the fast part of the spectrum. The experimental data, particularly those shown in Fig. 3, indicate that the spectra for the nuclei lie lower, in accord with the model described above. In addition, the representation of a collective statistical interaction with a tube may help explain the observed [13] exponential tail of the spectrum of the pions emitted backward, a tail that extends far beyond the threshold of the single-nucleon process.

In conclusion we note that the employed data are still insufficient for a reliable separation of different theoretical possibilities, particularly the hydrodynamic and parton models. More exact data are needed, especially at higher energies.

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Erratum: e^+e^- annihilation into hadron and collisions with nuclei in the hydrodynamic theory of multiple production

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1. Formula (2), the dependence of the multiplicity on the atomic number A, should read

$$N(A) = A^{\frac{1}{6} \frac{1+3c^2}{1+c^2}} = A^a$$

instead of the indicated $a = (1/6)(1-c^2)/(1+c^2)$, which corresponds to an initial volume equal to the nucleon volume and not to the tube volume (in the nucleon-tube rest system). The published value of a coincides with Gottfried's estimate^[9] based on the relation $N(A,S) = N(1,SA^{1/3})$, which is thus incorrect. The present estimate, with $c^2 = 1/3$, agrees with the result of [8a]. The author is grateful to I. L. Rozental', whose criticism

of [9] helped clarify this question.

- 2. The following misprints should be corrected: On p. 232, left column, 15th and 16th lines from the top, read "jet structure predicted by the parton model" instead of "ordered structure..." On page 232, right column, the reference number 171 should be added at the end of the last line of text.
- 3. The following addition should be made to the list of references:
- ¹⁵M.I. Atanelishvili, L. Berzenishvili, Yu.G. Verbetskii, et al., ZhETF Pis. Red. 19, 405 (1974) [JETP Lett. 19, 221 (1974)].