Damping of isobaric analog resonances

M. G. Urin

Moscow Engineering and Technical Institute
(Submitted March 13, 1975)
ZhETF Pis. Red. **21**, No. 8, 507–510 (April 20, 1975)

A formula that takes into account the connection between the proton channels of the decay of isobaric analog resonances (IAR) is obtained for the width of the "breakup" of the IAR on a configuration of complicated character as a result of the mechanism of "external" Coulomb mixing.

PACS numbers: 24.30.E, 23.90.

In view of the approximate isospin conservation, the width $\Gamma^{\downarrow}$ for the decay of an analog state (IAS) on a complex configuration exists to the extent of the Coulomb mixing of the indicated states, and is small on a nuclear scale ($\Gamma^{\downarrow} \ll 2w$, where w is the imaginary part of the optical potential at the excitation energy corresponding to the isobaric analog resonance (IAR)). If we neglect the influence of the isovector monopole resonance on the formation of the IAR, then the width $\Gamma^{\downarrow}$ is determined by the mechanism of "external" mixing, which is due to the existence of common (proton) IAR decay channels and of complicated configurations. The existing theoretical approaches to the description of IAR overestimate the width $\Gamma^{\downarrow}$ by several times in comparison with the experimental values.[1,2] One of the possible causes of this difficulty lies in the assumption that the proton channels of the IAR decay are independent. In this paper, the basis of the shell theory of the IAR,[3] a method is proposed to take into account the connection between the proton decay channels and is used to derive formulas for the IAR decay widths.

The correction $\delta\psi\nu$ to the wave function (WF) of the IAS, necessitated by the existence of the ν-th proton channel of IAR decay, is defined as the difference between the wave function $|a\rangle_0 = (2T_0+1)^{-1/2}T^{(-)}|n_\nu\rangle \equiv |d_\nu\rangle + (2T_0+1)^{-1/2}|p_\nu\rangle$, which is characterized by an isospin $T_0+1/2$ and contains a one-proton component $|p_\nu\rangle$ with the same spatial configuration as the state $|n_\nu\rangle$ of the parent nucleus, on the one hand, and the wave function $|a\rangle = |d_\nu\rangle + \sum_E a_{\nu E}|p_E\rangle$ with indeterminant isospin, which contains a continuum of one-proton states, on the other. The coefficients $a_{\nu E}$ are determined by the matrix elements of the isovector part of the shell potential $\mathbf{tT}V(r)$ between the states $|d_\nu\rangle$ and $|p_E\rangle$, namely $a_{\nu E} = (2T_0+1)^{-1/2}(T_0V)_{\nu E}(E_\nu - E + i\epsilon)^{-1}$. Consequently, in the coordinate representation the correction $\delta\psi_\nu$ is given by

$$\delta\psi_\nu(r) = -\int G_\nu(r, r')T_0V(r')\psi_\nu(r')dr' - \psi_\nu(r). \tag{1}$$

Here $\psi_\nu \equiv (2T_0+1)^{-1/2}\chi_\nu$; χ_ν is the radial wave function of the neutron belonging to the neutron excess in the parent nucleus: $(h_n - \epsilon_\nu)\chi_\nu = 0$, where $h_n = K + U + T_0V/2$ is the Hamiltonian of the shell model for neutrons; $G_\nu = \sum_E(E - E_\nu + i\epsilon)^{-1}\chi_E(r)\chi_E(r')$ is the Green's function; $(h_p - E)\chi_E = 0$, where $h_p = K + U - T_0V/2 + V_C$ is the shell-model Hamiltonian for the proton; K is the kinetic energy; U is the isoscalar part of the shell Hamiltonian; V_C is the average Coulomb energy of the proton-nucleus interaction; E_ν is the IAR energy in the photon channel. If the symmetry energy $T_0V \equiv v$ is replaced by the difference $v = h_n - (h_p - V_C)$, then the correction (1) can be represented in the form

$$\delta\psi_\nu(r) = -\int G_\nu(r, r')\{V_C(r') - E_\nu + \epsilon_\nu\}\psi_\nu(r')dr'. \tag{2}$$

Thus, the quantities (1) and (2) are generalized to include allowance for the continuous spectrum of the corrections to the wave functions of IAS as a result of the Coulomb mixing with antianalog states. In accordance with the foregoing conclusion, these corrections were obtained in the approximation in which the proton channels of the IAR decay are independent.

Depending on the size of the corrections (1), which violate the isotopic symmetry of the IAR, decay of IAS into complicated configurations $|\lambda\rangle$ are possible (these configurations are characterized by an isospin $T_0 - 1/2$), as a result of the "residual" nuclear interaction H'. The corresponding width is determined with allowance for the contribution of all the inelastic channels: $\Gamma^{\downarrow} = \sum_\nu \Gamma_\nu^{\downarrow}$. The partial width Γ_ν is the lowest order in H' is equal to

$$\Gamma_\nu^{\downarrow} = 2\pi\rho g_\nu |\langle\lambda|H'|\delta\psi_\nu\rangle|^2_{a\,\nu} = 2g_\nu\int w(r)|\delta\psi_\nu(r)|^2 dr. \tag{3}$$

Here ρ is the density of the configurations $|\lambda\rangle$; g_ν is a statistical factor; w is the imaginary part of the optical potential for protons of energy E_ν.

We take into account the connection between the photon channels of the IAR decay in the following manner. Let F be the intensity of the charge-exchange part of the effective nucleon interaction ($\hat{F} = (1/2)F\vec{\tau}_1 \cdot \vec{\tau}_2\delta(\mathbf{r}_1 - \mathbf{r}_2)$). Then the additional charge-exchange field (on top of the symmetry energy) acting on the nucleon during the course of the IAR proton decay is equal to

$$\delta v(r) = F(2T_0 + 1)\sum_\nu g_\nu(4\pi r^2)^{-1}\delta\bar{\psi}_\nu(r)\psi_\nu(r), \tag{4}$$

where the effective correction to the wave function of the IAS due to the ν-th proton channel of the $\delta\bar{\psi}_\nu$ decay is determined in turn by the field $v + \delta v$, i.e., by formula (1′), which is obtained from (1) by the substitution $\nu \to v + \delta v$. Thus, the self-consistent conditions (1′) and (4) make it possible to find the quantities $\delta\bar{\psi}_\nu$ and δ_ν. Since the relative difference between the corrections $\delta\psi_\nu$ and $\delta\bar{\psi}_\nu$ have no formal small parameter, we can expect this statement to be valid for the widths $\Gamma_\nu^{\downarrow}$(3) and $\bar{\Gamma}_\nu^{\downarrow}$:

$$\bar{\Gamma}_\nu^{\downarrow} = g_\nu\int w(r)\{|\delta\bar{\psi}_\nu[v + \delta v^*]|^2 + |\delta\bar{\psi}_\nu[v + \delta v]|^2\}dr. \tag{3'}$$

The need for symmetrization in this formula is brought about by the non-Hermitian character of δv.

For comparison with the experimental values of the widths $\Gamma^{\uparrow}$, formulas (3) and (3′) must be generalized to the case when account is taken of the effects nonlinear in $\Delta h = \Delta - iw$ (Δ is the change in the real part of the optical potential for protons in comparison with U). Thus, allowance for the excitation of the complex configurations during the course of the IAR proton decay leads to the following modification of the corrections (1) [3]:

$$\delta\tilde{\psi}_\nu(r) = \delta\psi_\nu(r) - \int \tilde{G}_\nu(r, r')\Delta h(r')\delta\psi_\nu(r')\,dr', \qquad (5)$$

where $\tilde{G}_\nu$ is the Green's function of the optical-model Hamiltonian for the protons: $\tilde{h}_p = h_p + \Delta h$. A similar equation holds also for the function $\delta\tilde{\tilde{\psi}}_\nu$. Then, using the method describes in[3], we obtain for the widths $\Gamma_\nu^{\uparrow}$ and $\bar{\Gamma}_\nu^{\uparrow}$ the expressions

$$\bar{\Gamma}_\nu^{\downarrow} + \bar{\Gamma}_\nu^{\uparrow} = \bar{\Gamma}_{\nu 0}^{\uparrow} - 2g_\nu \operatorname{Im} \int \delta\tilde{\psi}_\nu[v + \delta v^*]\Delta h(r)\delta\tilde{\tilde{\psi}}_\nu[v + \delta v]\,dr, \qquad (6)$$

where $\bar{\Gamma}_\nu^{\uparrow}$ and $\bar{\Gamma}_{\nu 0}^{\uparrow}$ are respectively the effective and "natural" protons widths of the IAR: $\bar{\Gamma}_{\nu 0}^{\uparrow} = \bar{\Gamma}^{\uparrow}[\Delta h = 0]$ and

$$\bar{\Gamma}_\nu^{\uparrow} = 2\pi g_\nu \left| \int \psi_\nu(r)(v + \delta v^* - \Delta h)_r \tilde{\chi}_{E_\nu}^{(+)}(r)\,dr \int \psi_\nu(r) \times (v + \delta v - \Delta h)_r \tilde{\chi}_{E_\nu}^{(+)}(r)\,dr \right| \qquad (7)$$

Here $(\tilde{h}_p - E)\tilde{\chi}_E^{(+)} = 0$. For concrete calculations, it is convenient to rewrite (6) in a form that does not contain the quantities $\delta\tilde{\psi}_\nu$ explicitly:

$$\bar{\Gamma}_\nu^{\downarrow} + \bar{\Gamma}_\nu^{\uparrow} = 2g_\nu \operatorname{Im}\left\{ \int \psi_\nu(v + \delta v^* - \Delta h)_r \tilde{G}_\nu(r, r')(v + \delta v - \Delta h)_{r'} \psi_\nu\,dr\,dr' - \int \psi_\nu^2 \Delta h\,dr \right\}. \qquad (8)$$

Direct verification shows that expressions (6) and (8) for the widths $\bar{\Gamma}_\nu^{\uparrow}$ go over into formula (3′) in first order in Δh.

[1]G. W. Bund and J. S. Blair, Nucl. Phys. **A144**, 384 (1970).
[2]N. Auerbach *et al.*, Rev. Mod. Phys. **44**, 48 (1972).
[3]M. G. Urin, Obolochechnye éffekty v rezonansnykh yadernykh reaktsiyakh s nuklonami (Shell Effects in Resonant Nuclear Reactions with Nucleons), MIFI (1974).