$SU(5)$ symmetry?

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It is suggested that the recently discovered particles $\psi_1(3105)$ and $\psi_2(3695)$ are members of a 25-plet of vector mesons within the framework of the $SU(5)$ groups. The masses of the I^- and 0^- mesons in the lowest multiplets of the $SU(5)$ group are calculated.

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Many papers published since the discovery[1–4] of the new neutral particles $\psi_1(3105)$ and $\psi_2(3695)$ discuss all possible approaches to the understanding of the nature of the new mesons. The most probable interpretation is considered to be the description of these states as bound quark-antiquark states with a "latent" new quantum number[5,6] within the framework of the symmetry group $SU(4)$. Then there is no place for the particle ψ_2 in the lowest multiplet of the I^- mesons, and it becomes necessary to assume that ψ_2 is a radially-excited state. In spite of all the known arguments in favor of this scheme, however, it cannot be said that it is satisfactory from all points of view. In particular, notice should be taken of the difficulty with explaining the ratio of the partial widths of the lepton decays of ψ_1 and ψ_2, let alone the absolute values of the width, namely, one would expect $\Gamma(\psi_2 \to l\tilde{l}) > \Gamma(\psi_1 \to l\tilde{l})$, whereas the experimental data point more readily to the inverse relationship.[1–4]

It is not our purpose, however, to discuss arguments for and against any one of the proposed approaches to the explanation of the new mesons. We wish only to note that, by starting from the experimental data on the properties of ψ_1 and ψ_2, there is room for the assumption that the discovery of these particles offers evidence of the existence of a strong-interaction symmetry group higher than $SU(4)$.

We introduce in this paper five fundamental quark fields. The three quarks p, n, and λ form the $SU(3)$ triplet, the fourth quark c differs in the charm quantum number,[7] and the fifth carries a new quantum number g. We regard ψ_1 and ψ_2 as almost pure states $c\tilde{c}$ and $g\tilde{g}$—in a strongly broken $SU(5)$ symmetry with approximately ideal mixing, and calculate the masses of the I^- and 0^- mesons in the lowest multiplets of $SU(5)$.

We write down the mass operator for the neutral vector mesons with $I = Y = 0$ in the form

$$M = \sum_{i,k=1}^{4} (A_i X_i^2 + X_i V_{ik} X_k),$$

where the basis states are

$$X_1 = \omega^0 = \frac{1}{\sqrt{2}}(p\tilde{p} + n\tilde{n}), \qquad X_2 = \phi^0 = \lambda\tilde{\lambda},$$

$$X_3 = \psi_1^0 = c\tilde{c}, \qquad X_4 = \psi_2^0 = g\tilde{g}.$$

and $A_2 = A_1 + \Delta\lambda$, $A_3 = A_1 + \Delta_c$, $A_4 = A_1 + \Delta_g$, and $A_1 = m_\rho$ (or m_ρ^2 if a quadratic mass operator is used); the terms Δ_λ, Δ_c, and Δ_g take into account the difference between the masses of the λ, c, and g quarks on one hand, and the masses of the p and n quarks on the other; the matrix V takes into account the deviation of the mixing

Masses of 1^- and 0^- mesons in the 25-plet of the $SU(5)$ group.

Quark configuration	Quadratic mass operator		Linear mass operator	
	$J^P = 1^-$	$J^P = 0^-$	$J^P = 1^-$	$J^P = 0^-$
$p\tilde{c}$	2.26	2.13	1.94	1.31
$\lambda\tilde{c}$	2.30	2.17	2.06	1.43
$p\tilde{g}$	2.66	2.54	2.28	1.65
$\lambda\tilde{g}$	2.70	2.59	2.40	1.77
$c\tilde{g}$	3.41	3.32	3.45	2.82
$c\tilde{c}$	3.105*	3.00	3.105*	2.48
$g\tilde{g}$	3.695*	3.61	3.695*	3.16

*Experimental values of the masses of the ψ_1 and ψ_2 mesons [1--4].

from ideal. The number of unknown parameters in the presented expression for the mass operator exceeds the number of known masses of the vector mesons. We must therefore make additional assumptions concerning the form of the matrix V. The experimental values of the partial widths of the hadron channels of the decay of the ϕ, ψ_1, and ψ_2 mesons suggest that the deviation of mixing from ideal is small, if the Izuko, Okubo, and Zweig rule is valid.[8,9] If the mixing deviates little from ideal, one choice of the matrix V or another changes the masses of the off-diagonal mesons in the multiplet insignificantly, and is reflected only in the values of the partial widths of the hadron channels of the neutral-meson decay.

To find the masses of the off-diagonal elements, both vector and pseudoscalar, we use therefore the simplest mass operator

$$M = \sum_{i=1}^{4} A_i X_i^2 + h(\sqrt{2}X_1 + X_2 + X_3 + X_4)^2,$$

which depends on the four parameters Δ_λ, Δ_c, Δ_g, and h. Assuming the deviation from ideal to be small, we neglect in the calculations the terms of order h^2. The table lists the calculated values of the masses of 1^- and 0^- mesons in the 25-plets of the $SU(5)$ group in the case of a linear and a quadratic mass operator. In the calculation of the masses of the pseudoscalar mesons we used for the parameters Δ_λ, Δ_c, Δ_g, and h values obtained from the masses of the neutral vector mesons:

$$\Delta_\lambda = 0.203\,\text{GeV}^2(0.122\,\text{GeV}), \quad \Delta_c = 4.50\,\text{GeV}^2\,(1.17\,\text{GeV}),$$

$$\Delta_g = 6.51\,\text{GeV}^2\,(1.51\,\text{GeV}), \quad h = 0.04\,\text{GeV}^2\,(0.005\,\text{GeV}).$$

The quantities in the parentheses are the parameter values obtained using a linear mass operator.

The observed states of ω, ϕ, ψ_1, and ψ_2, obtained with the quadratic mass operators, are expressed in terms of the ideal basis states in the following manner:

$$\omega = \omega^\circ + 0.1\,\phi^\circ + 0.006\,\psi_1^\circ + 0.004\,\psi_2^\circ\,,$$

$$\phi = -0.10\,\omega^\circ + \phi^\circ + 0.005\,\psi_1^\circ + 0.003\,\psi_2^\circ\,,$$

$$\psi_1 = -0.006\,\omega^\circ - 0.005\phi^\circ + \psi_1^\circ + 0.01\,\psi_2^\circ,$$

$$\psi_2 = -0.004\omega^\circ - 0.003\,\phi^\circ - 0.01\,\psi_1^\circ + \psi_2^\circ.$$

If a linear mass operator is used, the expressions deviate insignificantly from the foregoing.

The proposed $SU(5)$ scheme differs obviously from the $SU(4)$ model in that it predicts the existence of new mesonic and baryonic states ($p\tilde{g}, \lambda\tilde{g}, \ldots, ppg, p\lambda g, \ldots$) with quantum number g. The most massive baryon ggg in the lowest multiplet 3/2 has a mass M_{ggg} ~6 GeV. We note that in the case of a small deviation of the mixing from ideal, the masses of the baryons and mesons containing no g quarks differ insignificantly in the $SU(5)$ scheme from the masses predicted by the $SU(4)$ scheme.

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[1] J. E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974).
[2] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).
[3] C. Bacci *et al.*, Phys. Rev. Lett. 33, 1408 (1974).
[4] G. S. Abrams *et al.*, Phys. Rev. Lett. 33, 1453 (1974).
[5] A. De Rujula and S. L. Glashow, Preprint 18145, Harvard University (1974).
[6] T. Appelquist *et al.*, and S. L. Glashow, Preprint Harvard University (1974).
[7] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, 1285 (1970).
[8] J. Iizuka, Progr. Theor. Phys. Suppl., 37—39, 21 (1966).
[9] S. Okubo, Phys. Lett. 5, 165 (1963).