Mass formula for mesons and baryons with allowance for charm

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The mass formula previously proposed by Ya. B. Zel'dovich and the author for mesons and baryons is applicable to particles that have charm.

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The recent discovery of a vector particle ψ with mass 3105 GeV and with anomalously small hadronic-decay width, and its interpretation as consisting of charmed quarks,[1] make particularly essential an exhaustive analysis of the consequences of the hypothesis that a fourth quark exists. In this article we attempt to estimate the masses of mesons and baryons having charm with the aid of the mass formulas previously proposed by Ya.B. Zel'dovich and the author.[2] This formula is based on the "naive" model of nonrelativistic quarks and is written in unified form for mesons and baryons, namely additively in the masses,

$$\begin{aligned} M &= \delta + \Sigma m_q + b\Sigma \xi_i \xi_k (\vec{\sigma}_i \vec{\sigma}_k) \\ &= a + \Sigma(m_\lambda - m_0)|s| + \Sigma(m_\chi - m_0)|c| + b\Sigma \xi_i \xi_k (\vec{\sigma}_i \vec{\sigma}_k). \end{aligned} \qquad (1)$$

Here s is the strangeness, c is an analogous new additive quantum number, which we call in accordance with tradition "charm" and which is equal to the difference between the number of charmed quarks and antiquarks χ and $\bar{\chi}$, m_q is the mass of the quark, $m_q = m_0$ for the "ordinary" quarks p and n (O quarks). δ, b, and a are constants which are different for the meson and baryon. The last term describes the spin-spin interaction of the quarks; $\vec{\sigma}_i$ is the spin of the ith quark, ξ_i is the coefficient of attenuation of the spin-spin interaction, $\xi_\chi < \xi_\lambda < \xi_0 = 1$.

It turns out that the constants ξ_λ and the difference $m - m_0$ are approximately the same for baryons and mesons. For mesons, $\xi_\lambda = (K^* - K)/(\rho - \pi) = 0.645$ and

$$m_\lambda - m_0 = \frac{1}{4}(3K^* + K) - \frac{1}{4}(3\rho + \pi) = 194 \text{ MeV}.$$

For baryons we have

$$\xi_\lambda = 1 - \frac{3}{2}\,\frac{\Sigma - \Lambda}{\Delta - N} = 0.61, \quad m_\lambda - m_0 = \Lambda - N = 176 \text{ MeV}.$$

These coincidences may possibly indicate the presence of a physical meaning in formula (1).

To determine the constants pertaining to the charm quark, all we know for the time being is the mass of ψ. We assume this particle to consist of $\chi\bar{\chi}$, neglecting the mixing with the pairs $\lambda\bar{\lambda}$ and $O\bar{O}$. We make an additional assumption that connects the constants ξ with the masses of the quarks:

$$\xi_\lambda = \frac{m_0}{m_\lambda}, \quad \xi_\chi = \frac{m_0}{m_\chi}, \text{ т. е. } \xi_\lambda^{-1} = 1 + \frac{m_\chi - m_0}{m_\lambda - m_0}(\xi_\lambda^{-1} - 1). \qquad (2)$$

The constants entering in (1) are determined by the same token in a sufficiently unambiguous manner.

For numerical estimates, we assume also that the differences $m_\chi - m_0$ are different for mesons and baryons and are in the same ratio as the differences $m_\lambda - m_0$. We ultimately get the following system of constants:

TABLE 1.

	ϕ	Σ^*	Ξ	Ξ^*	Ω
Formula	1049	1375.5	1337	1520	1672
Experiment	1020	1385	1317	1530	1675

TABLE 2.

J	$O\bar{\chi},\ \bar{O}\chi$	$\lambda\bar{\chi},\ \bar{\lambda}\chi$	$\chi\bar{\chi}$
0	1748.5	1977.5	3076
1	1880.5	2062.5	-

TABLE 3.

J	$OO\chi$		$O\lambda\chi$			$\lambda\lambda\chi$	$O\chi\chi$	$\lambda\chi\chi$	$\chi\chi\chi$
	$\Sigma_c,\ \Sigma^*_c$	Λ_c	Ξ'_c	Ξ^*_c	Ξ''	Ω_c	N_{cc}	Ω_{cc}	Ω_{ccc}
1/2	2239.5	2078	2313		2404	2575	3331.5	3522.5	—
3/2	2298	–		2450.5		2611	3390	3559.0	4514

For mesons:

$a = 597$ MeV,
$b = 613$ MeV,
$m_\lambda - m_0 = 194$ MeV,
$m_\chi - m_0 = 1250.5$ MeV,
$\xi_\lambda = 0.645$,
$\xi_\chi = 0.216$.

For baryons

$a = 1088.5$ MeV,
$b = 200.0$ MeV,
$m_\lambda - m_0 = 176$ MeV,
$m_\chi - m_0 = 1140$ MeV,
$\xi_\lambda = 0.61$,
$\xi_\chi = 0.195$.

We shall explain the calculation of the third term of formula (1), using as an example baryons containing the three different quarks O, λ, and χ. The operator $H_{\sigma\sigma}$

$$H_{\sigma\sigma} = A(\vec{\sigma}_1\vec{\sigma}_2) + B(\vec{\sigma}_2\vec{\sigma}_3) + C(\vec{\sigma}_3\vec{\sigma}_1)$$

(where $A = b\xi_1\xi_2$ etc.) has eigenvalues

$$E_1 = \frac{A+B+C}{4} \quad \text{(spin 3/2)}$$

$$E_{2,3} = -\frac{A+B+C}{4} \pm \sqrt{A^2+B^2+C^2-AB-BC-CA} \quad \text{(spin 1/2)}.$$

The eigenvalues for spin 1/2 were obtained from the two-dimensional secular equation, which can be easily set up by recognizing that the three operators $(\vec{\sigma}_1\vec{\sigma}_2)$, $(\vec{\sigma}_2\vec{\sigma}_3)$, and $(\vec{\sigma}_3\vec{\sigma}_1)$ have the same eigenvalues and are obtained from one another by rotation through 120° in a two-dimensional plane.

The three eigenvalues correspond to three particles Ξ'_c, Ξ''_c, and Ξ^*_c (isodoublets) in Table 3. The remaining cases are even simpler.

The masses of the mesons π, ρ, K, K^* and ψ and of the baryons N, Δ, Σ, and Λ, with the assumed system of constants, are satisfied identically. For the meson ϕ and the baryons Σ^*, Ξ, Ξ^* and Ω formula (1) yields mass values which are very close to the experimental ones (Table 1).

In Table 2 are gathered the predictions of formula (1) for mesons containing one or two charmed quarks, and in Table 3 are gathered the formulas for baryons containing 1, 2, or 3 charmed quarks.

We note that the mass of the pseudoscalar meson O^-_χ (which we shall call K_c) predicted by the lines mass formula (1) is such that the decay $\psi' \to K_c + \bar{K}_c$ is possible. Here ψ' is a second vector meson with an anomalously small hadron-decay width, the mass of which is 3.7 GeV. No such decays have been observed at the present time. The predicted mass of the pseudoscalar meson $\chi\bar{\chi}$ was calculated without allowance for the mixing with χ and η.

Thus, the masses of five mesons and 13 baryons were predicted with the aid of the linear formula.

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[1]Ref. TH 1964-CERN, 6 December 1974.
[2]Ya. B. Zel'dovich and A. D. Sakharov, Yad. Fiz. 4, 395 (1966) [Sov. J. Nucl. Phys. 4, 283 (1967)].