Observation of the effect of cross section similarity in $\pi^- p(d)$ collisions with maximum momentum transfer to the target particle

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The behavior of different cross sections in $\pi^- p(d)$ collisions with maximum momentum transfer is
compared in the momentum interval 1.6–16 GeV/c. It is shown that the ratio of the partial cross sections
is on the average independent of the energy, and the effect of cross-section similarity appears in the
behavior of the cross sections themselves.

A study of different reactions with maximum momentum transfer (when the particle emitted forward is not identical with the incident particle) yields information on the most "profound" properties of strong interactions. We attempt to present a general description of the behavior of the cross sections of such reactions for $\pi^- p(d)$ interactions with maximum momentum transfer to the proton (deuteron). We consider the energy dependence of the reaction

$$\pi^- p \rightarrow p X^- \tag{1}$$

which includes elastic backscattering, backward resonance production, and production of the nonresonant part of the spectrum. It was noted earlier[1] that the inclusive cross section of this reaction in the momentum interval $1.64-3.25$ GeV/c decreases with increasing energy on the average at the same rate as the elastic backscattering.[2] It was also noted that the nonresonant part of the inclusive cross section in the interval $1.64-3.25$ GeV/c decreases with the momentum of the primary particle much more rapidly than the cross section for the backward ρ^--meson production.[1] We have analyzed the energy behavior of the cross sections of the reaction (1) on the basis of the available published data. Figure 1 shows the dependence of the differential elastic backscattering cross section $d\sigma/du'$

1 – V.V. Vladimirskii et al. (16th Conference on High-Energy Physics, Batavia, 1972). 2 – S.W. Kormanyos et al. (Phys. Rev., 164, 1661). 3 – E.S. Meanly et al. (Phys. Rev., D 6, 740). 4 – D.P. Owen et al. Phys. Rev., 181, 1794). 5 – W.F. Baker et al. (P.L. 28B, 291). 6 – E.W. Anderson et al. (Phys. Rev. Lett. V 22, 102). 7 – P.S. Aplin (Rep. Progr. Phys./H/44).

B.M. Abramov et al. Preprint ITEF-35, 1973.
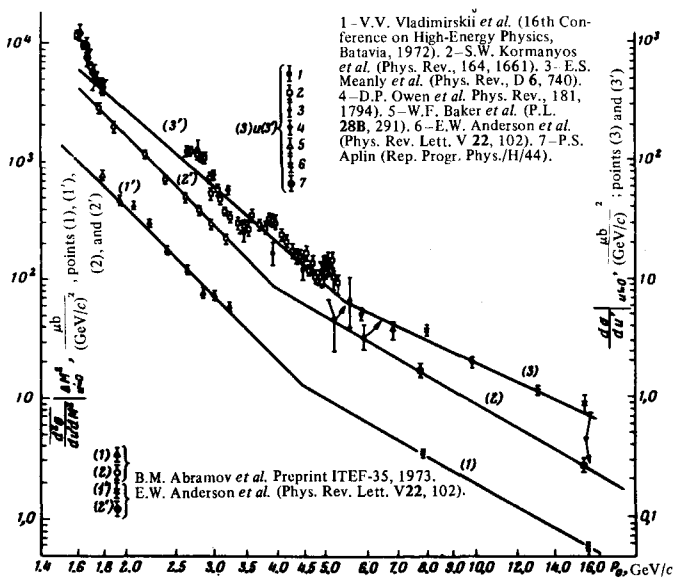E.W. Anderson et al. (Phys. Rev. Lett. V22, 102).

FIG. 1. Dependence of the differential cross section $d\sigma/du'$ of elastic π^-p backscattering on the momentum P_0 of the incident π^- meson—(3) and (3'). Dependence of the differential cross section of the inclusive process of the reaction $\pi^-p \to pX^-$ through the c.m.s. angle 180° on the momentum P_0 of the incident π^- meson; (1) and (1')—$(d^2\sigma/du'\,dM^2)$ for $\Delta M^2 = 0.2-0.45$ $(\text{GeV}/c^2)^2$; (2) and (2')—$(d^2\sigma/du'\,dM^2)$ for $\Delta M^2 = 0.2-0.95$ $(\text{GeV}/c^2)^2$. The straight lines were drawn by least squares.

and of the differential cross section of the inclusive process $(\overline{d^2\sigma/du'\,dM^2})\Delta M^2$ for two mass intervals, $\Delta M^2 = 0.2-0.45$ GeV^2 and $0.2-0.95$ GeV^2. All the employed data were taken at $u' = 0$ or close to zero, where $u' = u_{\text{max}} - u$, u is the square of the 4-momentum transfer, and u_{max} is the maximum kinematically permissible value of this momentum transfer. The principal data used in Fig. 1 were taken by us from[1-5]. It is seen from the figure that the energy dependence of the cross sections can be described by an expression of the form AP_0^{-n}, where P_0 is the momentum of the incident π^- meson. For elastic scattering in the interval $1.6-5.0$ GeV/c, the constants of the approximating expression AP_0^{-n} were taken from[2] and correspond to the so-called "nonresonant" elastic scattering at $u' = 0$. For convenience in the construction of Fig. 1, the points corresponding to the elastic-scattering cross section (3') in the interval 1.8 GeV/$c \leqslant P_0 \leqslant 2.7$ GeV/c are not shown, since they do not contain information of interest to us. In this interval there is a deep dip, due to the interference of the amplitudes of the "nonresonance" elastic scattering and the production of an isobar with mass 2.190 GeV. The remaining straight lines of Fig. 1 were drawn by least squares. The exponents n for the elastic and inclusive cross sections in different energy intervals agree within the limits of errors, and their values are 4 for 1.6 GeV/$c \leqslant P_0 \leqslant 5$ GeV/c and 2.5 for 5 GeV/$c \leqslant P_0 \leqslant 16$ GeV/c. For the sake of brevity, we shall henceforth designate by C_1, C_2, and C_3 the inclusive cross section $(d^2\sigma/du'\,dM^2)\Delta M^2$ averaged over the intervals $\Delta M^2 = 0.2-0.45$, $0.2-0.95$, and $0.45-0.7$ GeV^2, respectively.

Figure 2 shows the ratio of the elastic backscattering cross section and the inclusive cross section C_1. When

using C_1 and C_3 to calculate the ratios, we employed the experimental cross sections in the P_0 interval from 1.64 to 3.25 GeV/c and at 8 and 16 GeV/c.[1,3] At the intermediate points we used interpolated data (see Fig. 1). It is seen from the presented data that whereas the elastic and inclusive cross sections change by more than two orders of magnitude in the interval from 1.64 to 16 GeV/c, their ratio is on the average independent of the energy starting already with 1.64 GeV. It is typical that the exponent n changes for both cross sections at approximately the same energy $4.5-5.5$ GeV. It appears that in this interval a change occurs in the dominant exchange singularity in the u channel. The points in the interval $T_0 = 1.9-2.7$ GeV/c in Fig. 2 correspond to a deep dip in the energy dependence of the elastic scattering,[2] due to the production of the isobar Δ_{2190}, and was not taken into account in the analysis of the average dependence of the cross-section ratio on the energy.

Figure 3 shows the dependence of the ratio of the cross section for the production of the π^- meson[3,6,9] to the inclusive cross section C_3. It is seen from the diagram that with increasing energy the cross section ratio first increases, but at approximately 8 GeV the growth slows down and the ratio approaches a constant value. A similar behavior is possessed by the ratio of the π^--meson production cross section to the cross section of the nonresonant background. It should be noted that in the case of forward π^--meson production the analogous cross section ratio decreases with increasing energy.

Thus, it is seen from the analysis of the behavior of the cross sections of reactions (1) that whereas the partial cross sections vary greatly with energy (by more than two orders of magnitude), the ratio of the cross sections is practically independent of the energy. This behavior of the cross sections can be called relative gauge invariance (or relative scaling).

As a result, it is possible to describe the behavior of the cross sections of reaction (1) by a single common expression

$$\frac{d\sigma_i}{du'} = a_i\,\psi(P_0, u').$$

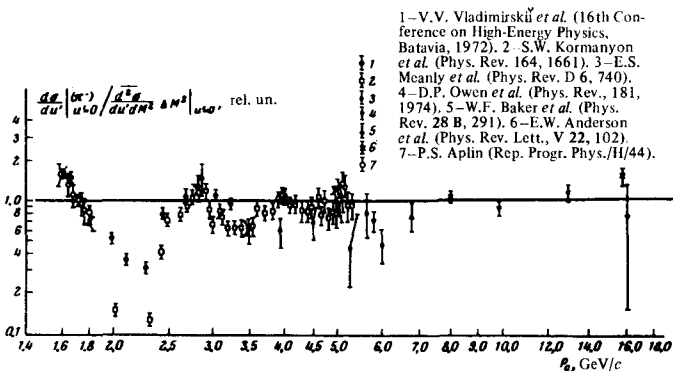Here $\psi(P_0, u')$ is a function common to all the channels

1 – V.V. Vladimirskii et al. (16th Conference on High-Energy Physics, Batavia, 1972). 2 – S.W. Kormanyon et al. (Phys. Rev. 164, 1661). 3 – E.S. Meanly et al. (Phys. Rev. D 6, 740). 4 – D.P. Owen et al. (Phys. Rev., 181, 1974). 5 – W.F. Baker et al. (Phys. Rev. 28 B, 291). 6 – E.W. Anderson et al. (Phys. Rev. Lett., V 22, 102). 7 – P.S. Aplin (Rep. Progr. Phys./H/44).

FIG. 2. Ratio of the differential cross section of elastic ^-p backscattering to the inclusive cross section $(d^2\sigma/du'\,dM^2)\,\Delta M^2$ of the reaction $\pi^-p \to pX^-$ (at $u' = 0$) vs. the momentum of the incident π^- meson. Here $\Delta M^2 = 0.2$ to 0.45 $(\text{GeV}/c^2)^2$. The straight line is drawn freehand. The circles and crosses denote points obtained using the experimentally measured inclusive cross section[1,3] while the remaining points were obtained with interpolated data.
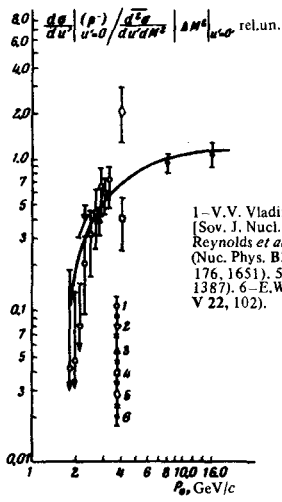
FIG. 3. Dependence of the ratio of the differential π^- meson production cross section to the differential inclusive cross section in the reaction $\pi^- p \to pX$ at $180°$ (c.m.s.) on the momentum of the incident π^- meson. Here $\Delta M^2 = 0.4-0.75$ $(\text{GeV}/c^2)^2$.

1—V.V. Vladimirskiĭ et al., Yad. Fiz. 17, 788 (1973) [Sov. J. Nucl. Phys. 17, 411 (1973)]. 2—R.C. Reynolds et al. (P.R. 184, 1424). 3—J.P. Baton et al. (Nuc. Phys. B21, 551). 4—L.J. Jonson et al. (Phys. Rev., 176, 1651). 5—B. Haber et al. (Phys. Rev., 10 D, 1387). 6—E.W. Anderson et al. (Phys. Rev. Lett., V 22, 102).

of the reaction (1), and a_i is a dimensional coefficient that is different for each ith channel of the reaction (1). For the inclusive cross section we imply

$$\frac{d\sigma_1}{du'} = \int_{M_1^2}^{M_2^2} \frac{d^2\sigma}{du'dM^2} \, dM^2 = f_1(M_1^2, M_2^2, P_o, u'),$$

where the parameters M_1^2 and M_2^2 (the limits of the averaging interval) are equal respectively to 0.2 and 0.95 GeV². The upper limit M_2^2 of the interval of averaging is determined by the conditions of our experiment at $P_0 = 1.68-3.25$ GeV/c.[11] In the considered momentum interval at $P_0 > 8$ GeV/c for the ρ^- meson and at 1.6 GeV/c $< P^0 < 16$ GeV/c for the remaining channels), the function $\psi(P_0, u')$ can be parameterized in the form

$$\psi(P_o, u') = P_o^{-n} e^{-B(P_o)u'}.$$

With this parametrization, the dimensionality of a_i is $\mu b (\text{GeV}/c)^{n-2}$, where n is the fall-off exponent, which is the same for all the considered channels of the reaction (1) (see Fig. 1). In[3-5,7-10] are given, for different channels of the reaction (1), the values of the so-called slope B corresponding to the universally accepted approximation of the dependence of the differential cross sections on u' in the form $d\sigma/du' = A \exp(-Bu')$. An analysis of the dependence of B on P_0 has shown that

Processes	Incident-particle moments		GeV/c
	1.24	1.48	1.68 ÷ 16.0
$\pi^- p \to pX^-$	–	–	0.932 ±0.19
$\pi^- d \to dX^-$	$0.37 \begin{smallmatrix} +0.17 \\ -0.07 \end{smallmatrix}$	$0.29 \begin{smallmatrix} +0.20 \\ +0.13 \end{smallmatrix}$	–

whereas the values of $B(P_0)$ change by more than a factor of two with increasing energy, the ratio $B(P_0)$ for different reactions to $B_1(P_0)$ for the inclusive process or for the production of the nonresonant background does not depend on the energy and is close to unity. The dependence of the slope $B(P_0)$ on P_0 can be parametrized in the form

$$B(P_0) = 4.75(1 - kP_0^{-0.8}),$$

where $k = 1(\text{GeV}/c)^{0.8}$.

Let us examine the behavior of the ratio of the elastic-scattering cross section to the inclusive cross section for another group of reactions. We have measured, at the momenta 1.24 and 1.48 GeV/c and at $u' \approx 0$, the cross sections of different reactions of the process

$$\pi^- d \to dX^- \qquad (2)$$

It turns out that the cross section of the process (2) is smaller, by almost three orders of magnitude, than the cross section of the process (1). If, however, we calculate the ratio of the cross section of the elastic $\pi^- d$ backscattering to the inclusive cross section, then this ratio is likewise independent of the energy and is close, as seen from the table, to the analogous ratio of the cross sections of their reactions of group (1).

Thus, our analysis shows that there exist groups of reactions [e.g., group (1) or group (2)], the cross sections of which depend in the same manner on the energy of the incident particles and on the momentum transfer, i.e., scaling is observed in the cross sections for definite groups of reactions.

[1]B. M. Abramov et al., Preprint ITEF-35 (1973).
[2]S. W. Kormanyos et al., Phys. Rev. 164, 1661 (1967).
[3]E. W. Anderson et al., Phys. Rev. Lett. 22, 102, 1390 (1969).
[4]V. V. Vladimirskiĭ et al., Paper at 16th Internat. Conf. on High-Energy Physics, Batavia, Ill, 1972.
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[7]B. M. Abramov et al., Preprint ITEF-34 (1973).
[8]B. M. Abramov et al., Preprint ITEF-72 (1973).
[9]J. P. Baton et al., Nucl. Phys. B21, 551 (1970).
[10]N. F. Baker et al., Phys. Lett. 28B, 291 (1970).