The problem of the magnetic moment of the deuteron

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It is shown that, when account is taken of the contributions made to the magnetic moment of the deuteron μ_D by the momentum-dependent terms of the local NN potential, it becomes possible to reconcile μ_D with experiment within the framework of the nonrelativistic Pauli theory. The magnetic moment of the deuteron then agrees well with diffraction D of the state and wave function of the deuteron $P_D = 5.5$–7%, and does not contradict any other data.

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There is a prevalent notion that the magnetic moment of the deuteron seemingly does not agree, within the framework of the Pauli nonrelativistic theory, with the fraction (P_D) of the D state in the wave function of the deuteron.

The part of μ_D which is due to the NN interaction is well known experimentally, namely $\Delta\mu_D = \mu_D - \mu_P - \mu_n = 0.022228 \pm 0.0001$ nuclear magnetons,[1] and it agrees with $P_D = 3.90 \pm 0.06\%$ when account is taken of only the kinematic contribution (due to $P_D \neq 0$) to $\Delta\mu_D$.

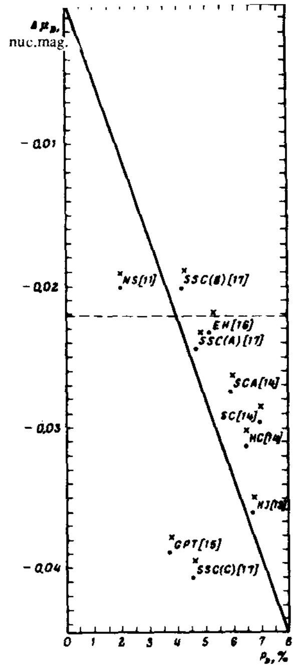
On the other hand, the quadrupole moment of the deuteron,[2] the form factor of the deuteron,[17] data on the photodisintegration of the deuteron,[4] data on the coherent production of pions on deuterium,[5] and the hyperfine splitting of the deuterium atom levels[6] seem to agree best with $P_D = 5.5-7\%$.

Attempts in 1957[7] and in 1973[8] to take into account, besides the kinematic contribution to $\Delta\mu_D$, also the contribution from the spin-orbit term of the NN potential, only made the situation worse, and led the authors to the conclusion that the Pauli nonrelativistic theory is not suitable for the description of the magnetic moment of the deuteron. However, the conclusion of[7] is based on calculations with the nonrelativistic potentials of[10] and[12], while the conclusion of[8] was decisively caused by an error incurred by the authors in the sign when calculating the spin-orbit contributions to $\Delta\mu_D$. In addition, in both calculations they used an incorrect formula for the spin-orbit contribution $\Delta\mu_D$, a fact pointed out in[3].

It will be shown below that allowance for the contributions made to $\Delta\mu_D$ from the momentum-dependent terms of certain contemporary realistic NN potentials makes it possible, within the framework of the Pauli nonrelativistic theory, to reconcile $\Delta\mu_D$ with $P_D = 5.5-7\%$. The values of the Dirac corrections are then ~$1/c^3$, they depend on the binding energy of the nucleons in the deuteron, and are relatively small for most NN potentials (see the figure).

The most general expression for the local NN-interaction operator, satisfying the general invariance principles and compatible with the Pauli equation, is obtained if we confine ourselves in the NN-potential operator to the second degree of the momentum operator:

$$\hat{V} = V^C(r) + V^T(r)\hat{S}_{12} + V^{LS}(r)\hat{\mathbf{L}}\hat{\mathbf{S}} + V^{(LS)^2}(r)\,(\hat{\mathbf{L}}\hat{\mathbf{S}})^2 + V^{L^2}(r)\hat{\mathbf{L}}^2 + \frac{1}{2m}\{v^{P^2}(r)\,\hat{\mathbf{P}}^2 + \hat{\mathbf{P}}^2 v^{P^2}(r)\} + \frac{1}{2m}\{(\hat{\vec{\sigma}}^{(1)}\hat{\mathbf{P}})\,v^{\sigma P}(r)(\hat{\mathbf{P}}\hat{\vec{\sigma}}^{(2)}) + (\hat{\vec{\sigma}}^{(2)}\hat{\mathbf{P}})\,v^{\sigma P}(r)(\hat{\mathbf{P}}\hat{\vec{\sigma}}^{(1)})\} + \frac{1}{2m}\{v^{TP^2}(r)\hat{S}_{12}\hat{\mathbf{P}}^2 + \hat{\mathbf{P}}^2\hat{S}_{12}v^{TP^2}(r)\}$$



Values of $\Delta\mu_D$ of the admixture D of the state in the wave function of the deuteron for certain NN potentials: points—values of $\Delta\mu_D$ within the framework of the Pauli theory, crosses—values of $\Delta\mu_D$ with allowance for the Dirac correction $\sim 1/c^3$. The numbers in the square brackets indicate the references, the letter indicates the variant of the potential in the notation of the corresponding reference, while the solid line shows the kinematic contribution made to $\Delta\mu_D$, equal to $-3/2(\mu_P+\mu_n-1/2)P_D$, dashed line—experimental value of $\Delta\mu_D$.

where

$$\hat{S}_{12} = (\hat{\vec\sigma}^{(1)}\mathbf{r})(\hat{\vec\sigma}^{(2)}\mathbf{r})/r^2 - 1/3\hat{\vec\sigma}^{(1)}\vec\sigma^{(2)}$$

$\hat\sigma_i^{(\nu)}$ are Pauli spin matrices acting on the spinors of the ν-th particle, $\mathbf{r}$ is the vector of the relative distance between the nucleons, $\hat{\mathbf{L}}$ is the operator of the relative orbital angular momentum of the pair of nucleons, $\hat{\mathbf{S}}$ is the operator of the total spin of the pair of nucleons, and m is the mass of the nucleon.

The Hamiltonian of the interaction with the electromagnetic field $\mathbf{A}$ was obtained by substituting $\partial/\partial r_P^\mu - i(e/\hbar c)A_\mu(r_p)$ for $\partial/\partial r_P^\mu$ in the initial Hamiltonian and by adding the Pauli term $-(\hat{\vec\mu}_P+\hat{\vec\mu}_n)$ curl $\mathbf{A}$ (here $\hat{\vec\mu}_P$ and $\hat{\vec\mu}_n$ are the operators of the proper magnetic moments of the proton and neutron, respectively, while r_P^μ is the coordinate of the proton).

After separating the motion of the mass center and averaging over the wave function of the deuteron, we obtain the following expressions for the mean values of the operators of the magnetic moments generated by the Pauli, kinetic, LS, $(LS)^2$, L^2, P^2, TP^2, and σP terms of the neutron-proton nonrelativistic Hamiltonian with the potential (1):

$$\mu_S = \mu_P + \mu_n - 3/2(\mu_P+\mu_n) < D \mid D > ,$$

$$\mu_K = 3/4 < D \mid D > ,$$

$$\mu_{LS} = 1/6 \{ < S \mid r^2 v^{LS} \mid S > - 1/\sqrt{2} < S \mid r^2 v^{LS} \mid D > - < D \mid r^2 v^{LS} \mid D > \},$$

$$\mu_{L^2} = 3/4 < D \mid r^2 v^{L^2} \mid D >,$$

$$\mu_{P^2} = 3/4 < D \mid v^{P^2} \mid D >,$$

$$\mu_{(LS)^2} = 1/\sqrt{2} < S \mid r^2 v^{(LS)^2} \mid D > + < D \mid r^2 v^{(LS)^2} \mid D >,$$

where

$$v^i = v^i/E_0;\ i = LS,\ (LS)^2,\ L^2;\ \ E_0 = \hbar^2/ma^2\Big|_{a=1_F} = 41.4686\,\text{meV};$$

$$< a \mid 0 \mid b > = \int_0^\infty \phi_a(r) 0(r) \phi_b(r) dr .$$

ϕ_S and ϕ_D are the radial wave functions of the deuteron and correspond to the orbital-momentum values $L=0$ and $L=2$, respectively, normalized by the condition $\langle S|S\rangle + \langle D|D\rangle = 1$, the calculated for the potential (1).

The considered contributions to the magnetic moment of the deuteron are all that are possible within the framework of the relativistic Pauli theory.

The dominant Dirac correction $\sim 1/c^3$ depends effectively on the deuteron binding energy E_D and takes the form

$$\Delta\mu_{\rm DIRAC} = \frac{1}{2}(\mu_P+\mu_n)E_D/mc^2 \ \text{nuc.mag.} \ [18].$$

The calculated values of $\Delta\mu_D$ shown in the figure indicate that for certain potentials the contribution from the momentum-dependent terms is so close to its experimental value, that by subjecting the spin-orbit potential to a change that is immaterial for all the other two-nucleon data it is possible to make the theoretical values of $\Delta\mu_D$ agree as closely as desired with experiment.

Another possibility of constructing a realisitic NN potential that describes arbitrarily accurately the magnetic moment of the deuteron lies in the existence of a group of unitary transformations of the NN potential, that change neither the NN-scattering phase nor those deuteron properties which are not connected with the interaction with the external field.[9] Under this transformation, the potential (1) retains its operator form, but all the radial functions that enter in it are changed, and this leads to a change in all the electromagnetic properties of the deuteron. This makes it possible to describe $\Delta\mu_D$ as accurately as desired by means of a corresponding unitary transformation of any realistic NN potential.

The magnetic moment of the deuteron therefore plays a rather exceptional role in the inverse problem of elastic NN scattering, greatly limiting the class of unitary transformations.[9]

The remaining freedom of transforming the even triplet component of the NN potential, and the total freedom of transforming the even singlet component, can be used to reconcile with experiment the magnetic moments, the binding energies, and other characteristics of such nuclei as ^{3}H and ^{3}He.

Summarizing, it must be emphasized that realistic NN potentials that are well fitted to all the two-nucleon data lead, within the framework of the nonrelativistic Pauli theory, to a satisfactory description of the magnetic moment of the deuteron, and this description can be made as accurate as desired without making use of any additional contributions to the magnetic moment of the deuteron.

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