Extensions of the Poincare algebra by spinor generators

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The structure of the possible nontrivial extensions of the Poincare algebra by arbitrary spinor generators is considered.

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Recently, considerable interest has arisen in symmetry algebras containing spinor generators. In[1-4] they considered extensions of the Poincare algebra by generators that transform in accordance with two-row spinor representations of the Lorentz group (the representations $(1/2, 0)$ and $(0, 1/2)$).

In this paper we consider the possible nontrivial extensions of the Poincare algebra by arbitrary spinor generators, i.e., algebras S that consist of generators of Lorentz transformations J_{AB} and $J_{\dot{A}\dot{B}}$ shifts $p_{A\dot{B}}$, and spinor generators $Q^{(j,k)}_{A_1\ldots A_{2j}\dot{B}_1\ldots\dot{B}_{2k}}$, which transform in accordance with the irreducible representations (j,k) of the Lorentz group ($j+k$ is a half-integer).[1)]

To preserve the correct connection between the spin and statistics, the algebra S should contain anticommutators between the spinor generators.

From the structure of the extensions of the direct product of the representations of the Lorentz group into direct sums it follows that in order for an algebra S to be a nontrivial extension of the Poincare algebra, it must contain at least two spinor generators $Q^{(j,k)}$ and $\tilde{Q}^{(j',k')}$, for which the relations $|j-j'|=1/2$ and $|k-k'|=1/2$ must be satisfied.

Let us consider the algebras $S^{(j)}$ containing the spinor generators $Q^{(j,k)}$ and $\tilde{Q}^{(k,j)}$, i.e., $Q^{(j,j-1/2)}$ and $\overline{Q}^{(j-1/2,j)}$).

From the generalized Jacobi identities it follows that only three following types of $S^{(j)}(j>1/2)$ algebras are possible (we do not write out the standard commutation relations for J and P):

I $$[P_{A\dot{B}}, Q^{(j,j-1/2)}_{A_1\ldots A_{2j}\dot{B}_1\ldots\dot{B}_{2j-1}}] = a \underset{(A_1,\ldots,A_{2j})}{Sym}\, \epsilon_{AA_1}\overline{Q}^{(j-1/2,j)}_{A_2\ldots A_{2j}\dot{B}\dot{B}_1\ldots\dot{B}_{2j-1}},$$

$$[P, \overline{Q}^{(j-1/2,j)}] = 0, \quad \{Q^{(j,j-1/2)}, Q^{(j,j-1/2)}\} = 0,$$

$$\{Q^{(j,j-1/2)}, \overline{Q}^{(j-1/2,j)}\} = 0, \quad \{\overline{Q}^{(j-1/2,j)}, \overline{Q}^{(j-1/2,j)}\} = 0.$$

II $$[P_{A\dot{B}}, \overline{Q}^{(j-1/2,j)}_{A_1\ldots A_{2j-1}\dot{B}_1\ldots\dot{B}_{2j}}] = a \underset{(B_1,\ldots,B_{2j})}{Sym}\, \epsilon_{\dot{B}\dot{B}_1} Q^{(j,j-1/2)}_{AA_1\ldots A_{2j-1}\dot{B}_2\ldots\dot{B}_{2j}}$$

$$[P, Q] = 0, \quad \{Q, Q\} = 0,$$

$$\{Q, \overline{Q}\} = 0, \quad \{\overline{Q}, \overline{Q}\} = 0,$$

III $$[P, Q] = 0, \quad [P, \overline{Q}] = 0, \quad \{Q, Q\} = 0, \quad \{\overline{Q}, \overline{Q}\} = 0,$$

$$\{Q_{A_1\ldots A_{2j}\dot{B}_1\ldots\dot{B}_{2j-1}}, \overline{Q}_{C_1\ldots C_{2j-1}\dot{D}_1\ldots\dot{D}_{2j}}\} =$$

$$= b \underset{(A,B,C,D)}{Sym}\, \epsilon_{A_1C_1}\ldots\epsilon_{A_{2j-1}C_{2j-1}}$$

$$\times\, \epsilon_{\dot{B}_1\dot{D}_1}\ \ \epsilon_{\dot{B}_{2j-1}\dot{D}_{2j-1}} P_{A_{2j}\dot{D}_{2j}},$$

where Sym denotes symmetrization with respect to the indices $\epsilon_{AB}=-\epsilon_{BA}$ and $\epsilon_{\dot{A}\dot{B}}=-\epsilon_{\dot{B}\dot{A}}$, while a and b are arbitary numbers. We leave out the spinor indices where there is no danger of misunderstanding.

In the case $j=1/2$, two types of algebra are possible:

I $$[P_{A\dot{B}}, Q_C] = a\epsilon_{AC}\overline{Q}_{\dot{B}}; \quad [P_{A\dot{B}}, \overline{Q}_{\dot{C}}] = 0,$$

$$\{Q_A, \overline{Q}_{\dot{B}}\} = bP_{A\dot{B}}, \quad \{Q_A, Q_B\} = abJ_{AB}, \quad \{\overline{Q}_{\dot{A}}, \overline{Q}_{\dot{B}}\} = 0.$$

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$$\ldots_{AB}, \ldots_{C}\ldots [illegible] \ldots_{AB}, Y_C\} = a\,\epsilon_{\dot B\dot C} Y_A,$$

$$\{Q_A, \bar{Q}_{\dot B}\} = bP_{A\dot B}, \quad \{Q_A, Q_B\} = 0, \quad \{\bar{Q}_{\dot A}, \bar{Q}_{\dot B}\} = abJ_{\dot A\dot B}.$$

We see that in this case not only $P_{A\dot B}$ but also J_{AB} (or $J_{\dot A\dot B}$) are bilinear combinations of the spinor generators.

We note that for all the permissible algebras $S^{(j)}$ we have $[P,(P,Q)]=0$. As a result, for any finite algebra that contains $S^{(j,j-1/2)}$ as a subalgebra and contains no other spinor generators, the O'Raifeartaigh generalized theorem, which is proved for the case $j=1/2$ in[5], is valid.

For $S^{(j,j-1/2)}$ algebras of type I and II, P^2 is not an invariant, and therefore only the algebra of type III (for $j=1/2$ and $a=0$) can be regarded as a possible symmetry algebra of the theories describing particle interactions. We present the fundamental results. The algebra III can be realized as an algebra of the group of transformations of a $4-4j(2j-1)$ dimensional superspace with coordinates $X_{A\dot B}$ and $\theta_{A_1\cdots A_{2j}\dot B_1\cdots\dot B_{2j-1}}$ and $\bar\theta_{C_1\cdots C_{2j-1}\dot D_1\cdots\dot D_{2j}}$ ($X_{A\dot B}$ are coordinates of Minkowskii space, and $\theta(\bar\theta)$ are anticommuting spinors):

$$\theta \to \theta + \xi, \qquad \bar\theta \to \bar\theta + \bar\xi,$$

$$X_{A\dot B} \to X_{A\dot B} - i\frac{b}{2}(1+f)\xi_{AA_2\ldots A_{2j}\dot B_1\ldots\dot B_{2j-1}}\bar\theta_{A_2\ldots A_{2j}\dot B\dot B_1\ldots\dot B_{2j-1}}$$

$$+\, i\frac{b}{2}(1-f)\theta_{AA_2\ldots A_{2j}\dot B_1\ldots\dot B_{2j-1}}\bar\xi_{A_2\ldots A_{2j}\dot B\dot B_1\ldots\dot B_{2j-1}},$$

where ξ is an anticommuting spinor and f is an arbitrary number.

In analogy with the case $j=1/2$, we introduce the superfields $\psi(X,\theta,\bar\theta)$, which transform in accordance with the representations of the considered group. The irreducible superfields are classified in accordance with the values of Y ($Y=0,\ 1/2,\ 1,\ 3/2,\ldots,$) and are equivalent to the set of the ordinary fields with a spin spectrum

$$Y-j(4j-1),\ Y-j(4j-1)+\frac{1}{2},\ldots,\ Y-\frac{1}{2},\ Y,\ Y+\frac{1}{2},\ldots,$$

$$Y+j(4j-1)-\frac{1}{2},\quad Y+j(4j-1).$$

Using the superfield technique we can, in analogy with the case $j=1/2$, construct a number of models that are invariant with respect to the considered group. These models are characterized by the presence of a large number of cancellation of the divergences, and also by the presence of fields (particularly gauge fields) with high integer and half-integer spins. Thus, the algebras $S^{(j)}$ with $j>1/2$ are of interest for the description of the interaction of fields with high spins.

We note in conclusion that we can consider in similar fashion algebras S that contain: (α) spinor generators with $j'\neq k$ and $k'\neq j$, (β) several spinor generators with different j and k, and (γ) spinor generators that transform in accordance with reducible representations of the Lorentz group (for example, $(1/2,\ 1/2)^n\otimes(1/2,0)$). In these cases, a number of interesting singularities are observed.

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1) We use spinor notation: $A,B=1,2$, $\dot A,\dot B=1,2$. The irreducible quantities (for example J and Q) are symmetrical with respect to the indices with and without the superior dots.

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