Coherent model of ψ bosons

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The interpretation of ψ particles as a boson condensate explains the observed suppression of the amplitudes of pure pionic channels of the decay of ψ in comparison with process $\psi' \to 2\pi\psi$.

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A distinguishing feature of ψ bosons is the strong suppression of the pure pionic decay channels, whereas the width for the electromagnetic decay into e^+e^- is of the same order of magnitude as for other vector mesons.

The main idea of this paper is that the ψ boson is a multiparticle system of the condensate type—a certain coherent state of the pionic field. In this model, the suppression of the decay into a finite number of pions is due to the fact that the norm of the state is distributed over an unusually large number of particles. On the other hand, the "in-phasing" of the components of the condensate should lead to the appearance of collective degrees of freedom interacting with the electromagnetic field. Consequently the decay $\psi \to e^+e^-$ can be no less probable than the annihilation of two charged hadrons into a lepton pair.

In this article we consider hadronic decays of ψ bosons. We assume for simplicity that the pions are neutrals and the ψ particles are spinless. We treat the ψ boson as a quasiparticle excitation of the pionic condensate. Then, if we start from the canonical Bogolybov transformation for boson operators, we can connect the ψ-boson annihilation operators b with the pion operators a and a^+:

$$b(\alpha) = e^{\alpha A_+} a\, e^{-\alpha A_+} = ua + va^+, \qquad A_+ = (a^+)^2/2,$$
$$u^2 - v^2 = 1, \qquad \alpha = v/u = \operatorname{th}\chi \tag{1}$$

(we have left out the indices that indicate the degrees of freedom of the field).

The normalized state vectors of the quasiparticle vacuum $|0,\alpha\rangle$ and of the single-quasiparticle excitation $|\alpha\rangle$ over this vacuum are given by the following formulas (see, e.g., [1]):

$$|0,\alpha\rangle = \operatorname{ch}^{-1/2}\chi\, e^{\alpha A_+}|0\rangle, \qquad |\alpha\rangle = \operatorname{ch}^{-3/2}\chi\, e^{\alpha A_+}|1\rangle. \tag{2}$$

Here $|0\rangle$ and $|1\rangle$ are states without pions and with one pion, respectively.

It is easy to verify that $|0,\alpha\rangle$ contains states with only even numbers of pions, and $|\alpha\rangle$ with only odd numbers. Thus, the G-parity of the state $|\alpha\rangle$, which is identified with the ψ particle, is negative. The average number of pions in the quasiparticle vacuum is

$$\bar{n} = \langle 0,\alpha|a^+a|0,\alpha\rangle = \alpha^2(1-\alpha^2)^{-3/2} \approx e^{3\chi} \tag{3}$$

(the last link of this equation is valid if $e^\chi \gg 1$).

States with different α, describing different quasiparticles, are not orthogonal. In particular:

$$\langle 0,\alpha|0,\alpha'\rangle = \operatorname{ch}^{-1/2}(\chi - \chi'). \tag{4}$$

Allowance for the spatial degrees of freedom of the field $\mathbf{k}$ (pion momentum) causes α to become a function of $\mathbf{k}$. The spectrum of the quasiparticles and the functions $\alpha(\mathbf{k})$ themselves should be determined by the equations for $\alpha(\mathbf{k})$, which follow from the dynamics of the field.

corresponding state vectors are direct products of the vectors $|0,\alpha(\mathbf{k})\rangle$ or $|\alpha(\mathbf{k})\rangle$ for all $\mathbf{k}$. Relations (3) and (4) go over into integrals with respect to $\mathbf{k}$ (the entire exponential is integrated in (3), and its argument is integrated in (4)). Assuming that these integrals converge, we shall use formulas (3) and (4) for estimates, with the understanding that they pertain to certain average momentum $\mathbf{k}$.

We assume that the bosons $\psi'(3.7)$ and $\psi(3.1)$ correspond to close values of the parameters χ' and χ. Then the invariant amplitudes for the decays $\psi' \to 3\pi$ and $\psi' \to 2\pi\psi$ can be written in the form

$$M_{3\pi} = g\sqrt{6}\langle 0|0,\alpha\rangle = g\sqrt{6}\,e^{-\chi^2}, \tag{5}$$

$$M_{2\pi\psi} = g'\sqrt{2}\langle 0,\alpha|0,\alpha'\rangle = g'\sqrt{2}\,\mathrm{ch}^{-1/2}(\chi-\chi'). \tag{6}$$

Here g and g' are constants of the corresponding effective four-boson interactions ($\pi^+\pi^+\pi^+\psi'$ and $\pi^+\pi^+\psi^+\psi'$). The meaning of these constants is that they take into account the difference between the "condensate" pions and the real ones, and therefore should be of the same order of magnitude. According to the experimental data, the width $\Gamma_{3\pi}$ is less than or comparable with the width $\Gamma_{2\pi\psi}$, whereas the invariant phase volume for the three-pion decay is 260 times larger than for the $\psi' \to \psi$ transition. On this basis, and assuming $g \approx g'$ and $\chi \approx \chi'$, we easily obtain

$$\chi \gtrsim 7, \qquad \bar{n} \gtrsim 10^9.$$

In this model, the large number of particles in the condensate is due to the fact that the suppression of the transition from the quasiparticle vacuum to the pion vacuum is $\sim\bar{n}^{-1/3}$. If we use for the description of the ψ particles the usual (oscillator) coherent states (see[2]), then the suppression of the pure pionic modes of ψ decay will be exponential in $\bar{n}$. In this case the transition probabilities of the "coherent vacuums" into each other and into the ordinary one will be

$$|\langle 0,\beta|0,\beta'\rangle|^2 = e^{-|\beta-\beta'|^2}, \qquad |\langle 0|0,\beta\rangle|^2 = e^{-|\beta|^2}, \tag{7}$$

with the parameter β connected with the average particle number by

$$\bar{n} = |\beta|^2. \tag{8}$$

Regarding ψ and ψ' as excitations over the vacuum $|0,\beta\rangle$ and using formulas of the type (5) and (6) (with α replaced by β and with the matrix elements (3) and (4) replaced by (7) and (8)), we obtain

$$\bar{n} \gtrsim 10.$$

Direct use of the oscillator and coherent states for the ψ particles is made formally difficult by the fact that the latter have negative G parity. We note also that in the case of oscillator coherent states the ψ particles should be described by solutions of the soliton type rather than the quasiparticle type. The particles ψ and ψ' should be regarded in this case as quantized small oscillations about an average field of the soliton type (of the kind considered in[3]), corresponding to the "coherent vacuum" state $|0,\beta\rangle$. However, the main cause of the suppression of the pion decays of ψ in all such models is the same—the difficulty of restructing into a system with a finite number of degrees of freedom. This is determined quantitatively by the factor $|\langle 0|0,\alpha\rangle|^2$. Obviously, the cross section of any reaction with absorption or production of ψ will contain this factor, and should therefore be small in comparison with the ordinary hadronic cross sections.

The considered possibility of describing ψ particles is of interest because it makes it possible to get along without introducing new hypothetical quantum numbers. We note that recent progress in the theory of coherent states[4] has greatly broadened the scope of theoretically conceivable coherent models, while confining itself to that group of ideas which are connected with nonlinear field Hamiltonians that leads to spontaneously broken symmetries.

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