Quasienergy distribution of electrons interacting with optical phonons in an electric radiation field

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(Submitted May 3, 1975)
ZhETF Pis. Red. 21, No. 11, 629–633 (June 5, 1975)

We investigate the stationary distribution of nondegenerate electrons interacting with zero-point optical oscillations of a crystal, of frequency ω_0, in the presence of strong radiation. We describe the singularities that arise at definite frequencies Ω of the light: 1) photon-phonon resonance ($\Omega = n\omega_0$); 2) discrete distribution of the electrons with respect to the quasienergies at $\Omega = [p + (n/m)\omega_0,\ n < m]$; 3) cooling of the electrons ($\Omega < \omega_0$).

PACS numbers: 63.20.K

1. A strong electric radiation field in which electrons oscillate ($\Omega\tau \gg 1$, where τ is the electron relaxation time) changes the interaction between the electrons and the phonon thermostat, so that distribution over the quasienergies differs significantly from equilibrium.[1] The distribution function $f(\epsilon)$, under the condition

$$\gamma \equiv \frac{1}{6\hbar\Omega}\frac{(eE\Omega^{-1})^2}{m^*} << 1 \tag{1}$$

(m^* is the effective mass of the electron, E is the amplitude of the light wave), which makes it possible to neglect multiphoton processes, is determined from the equation

$$\delta\kappa\frac{d}{d\epsilon}\left[b(\epsilon,\epsilon)\left(t\frac{df}{d\epsilon}+f(\epsilon)\right)\right] + [a(\epsilon,\ \epsilon+1)f(\epsilon+1)$$
$$- a(\epsilon,\epsilon-1)f(\epsilon)] + \gamma[b(\epsilon,\ \epsilon+\omega+1)f(\epsilon+\omega+1)$$
$$+ b(\epsilon,\ \epsilon-\omega+1)f(\epsilon-\omega+1) - b(\epsilon,\epsilon+\omega-1)f(\epsilon)$$
$$- b(\epsilon,\epsilon-\omega-1)f(\epsilon)] = 0, \tag{2}$$

where $a(\epsilon,\epsilon') = \sqrt{\epsilon\epsilon'}\theta(\epsilon')$, $b(\epsilon,\epsilon') = \sqrt{\epsilon\epsilon'/\omega}(\epsilon+\epsilon')\theta(\epsilon')$, $t = T/\hbar\omega_0$, $\omega = \Omega/\omega_0$, $\delta = (D_{ac}/D_{opt})^2 t$, $\kappa = (2m^*s^2/T)\omega \ll 1$, D_{ac} and D_{opt} are respectively the acoustic and optical deformation potentials, T is the lattice temperature, s is the speed of sound; the dimensionless quasienergy of the electron ϵ is measured in units of $\hbar\omega_0$. The first term in the left half of (2) describes the change of $f(\epsilon)$ due to quasielastic scattering by acoustic phonons, the second describes the change due to emission of optical phonons (it is assumed that $t \ll 1$), while the last term accounts for the interaction with the light when optical phonons are emitted. Equation (2) was obtained from the quantum kinetic equation,[2,3] using (1) and the condition $\delta \ll 1$, which makes it possible to neglect the interaction with light when acoustic phonons participate. Since usually $D_{ac} \sim D_{opt}$, the smallness of δ follows from the assumed smallness of t.

Owing to the intense emission of optical phonons at $\epsilon > 1$, the function $f(\epsilon)$ is small in that case like γ, and can be simply calculated in terms of its values in the interval $(0, 1)$, in which

$$\frac{d}{d\epsilon}\left[b(\epsilon,\epsilon)\left(t\frac{df}{d\epsilon}+f(\epsilon)\right)\right] - ab(\epsilon,\ \epsilon+\omega-1)f(\epsilon)$$
$$+ ab(\epsilon-\omega+p,\ \epsilon+p-1)f(\epsilon-\omega+p)\theta(\epsilon-\omega+p)$$
$$+ ab(\epsilon-\omega+p+1,\ \epsilon+p)f(\epsilon-\omega+p+1)\theta(\omega-p-\epsilon) = 0, \tag{3}$$

where $a = \gamma/\kappa\delta$ and $p = 1, 2, \cdots$, is the integer part of ω. Equation (3) is solved under the condition that there is no flux, $j(\epsilon) = -b(\epsilon,\epsilon)[t(df/d\epsilon) + f(\epsilon)] = 0$, as $\epsilon \to 0$ and under the condition $f(1) = 0$ ($\sim\gamma$ or $\sim e^{-1/t}$) as $\epsilon \to 1$; $f(\epsilon)$ is always normalizable in this case.

2. At the exact equality $\omega = n = 1, 2, \cdots$, it follows from (3) that $f(\epsilon) = c\exp(-\epsilon/t)$, i.e., the light field does not influence the electron distribution over the quasienergies. At a small frequency deviation $\omega' = \omega - n > 0$ (such that $\omega'|df/d\epsilon| \ll f$), Eq. (3) in which one should put $p = n$, has in the entire interval $(0, 1)$, with the exception of the narrower layer $(0, \omega')$, the first integral

$$b(\epsilon,\epsilon)\left(t\frac{df}{d\epsilon}+f(\epsilon)\right) - a\omega' b(\epsilon,\ \epsilon+n-1)f(\epsilon) = -C. \tag{4}$$

Equation (4) has a solution that vanishes at $\epsilon = 1$ for the lower part of the considered interval $\epsilon < \epsilon_2$, where ϵ_2 is determined from

$$\int_{\epsilon_2}^{1}\left(a\omega'\frac{b(\epsilon,\ \epsilon+n-1)}{b(\epsilon,\epsilon)} - 1\right)d\epsilon = 0$$

is essentially non-Maxwellian

$$f(\epsilon) = \frac{C}{a\omega' b(\epsilon,\ \epsilon+n-1) - b(\epsilon,\epsilon)}, \qquad n \geqslant 2, \tag{5}$$

and decreases rapidly at $\epsilon > \epsilon_2$ (like a Maxwellian function). With increasing $\alpha\omega'$, the energy ϵ_2 increases, and at $\alpha\omega'[b(1,n)/b(1,1)] \geqslant 1$ the electron distribution is described by formula (5) in practically the entire interval $(0, 1)$. At $\omega' < 0$, Eq. (3), in which we now must put $p = n - 1$ and $n \geqslant 2$, has the first integral (4) practically everywhere except in the narrow layer $(1 - |\omega'|, 1)$. A solution of (4) that satisfies the condition $j(0) = 0$ at $\omega' < 0$, starting with the smallest $\alpha|\omega'|$, is given approximately by formula (5) with $C < 0$, so that the change of $f(\epsilon)$ following the appearance of a frequency detuning does not take place gradually (as when $\omega' > 0$), but jumpwise.

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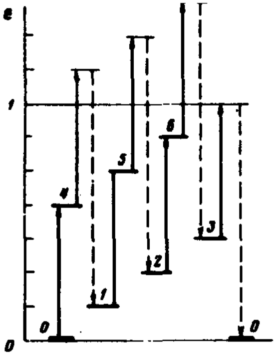

Arrangement of quasilevels $f(\epsilon)$ for $n/m=4/7$. The transition at $\omega=p+(n/m)$ (with emission of p-phonons) is represented by the vertical solid line. The dashed line represents the emission of the additional phonon.

The stationary weak-field conductivity σ of a semiconductor situated in a strong optical field is calculated in the usual manner in terms of the function $f(\epsilon)$.[4] Near $\omega=n$, the conductivity σ exhibits (as do other kinetic coefficients) a strong resonant dependence on ω' (photon-phonon resonance). When the momentum is scattered by acoustic phonons, the transition from the Maxwellian $f(\epsilon)$ (with $t \ll 1$) to the function (5) is accompanied by a decrease of the mobility by a factor $\sim t^{-1/2}$, so that the photon-phonon resonance should become manifest in σ by sharp peaks at $\omega=n \geqslant 2$, which decrease rapidly on the red side and more smoothly on the violet side.

3. The case $n=1$ is singular, since the frequency $\omega=1$ is nonresonant. As seen from (4) (which can be used on both sides of $\omega=1$ at small values of $|\omega'|$), in the vicinity of this frequency $f(\epsilon)$ is almost Maxwellian with a temperature $t'=t/(1-\alpha\omega')$, i.e., the electrons become heated at $\omega'>0$ and cooled at $\omega'<0$. The latter is typical of all frequencies $\omega<1$, since absorption of a photon of this frequency is accompanied by emission of a phonon with higher energy. For the cooled electrons, $f(\epsilon)$ can be obtained from (3), which is valid at $\omega<1$ if we put in it $p=0$ and omit the next-to-last term in the left-hand side. In stronger fields, $\gamma \gg t\kappa\delta(1-\omega)^2$, all the electrons go over into the region $\epsilon<1-\omega$, where $f(\epsilon)$ is Maxwellian with a temperature t. At $1-\omega<t$ and $1 \gg t \gg \kappa\delta$, the cooling is strong. The described situation differs from other models, in which absolute cooling of the electrons is predicted.[5-8]

Cooling is possible also at $\omega'>0$, if $\alpha \gg 1$ and $\alpha\omega' \gtrsim 1$. This region is characterized by essential singularities of σ.

4. If $\omega=p+1/2$, then (3) is transformed into a system of equations for the functions $f(\epsilon)$ and $f_1(\epsilon)=f(\epsilon+1/2)$ in the interval $(0,1/2)$:

$$\frac{dj(\epsilon)}{d\epsilon} = -\frac{dj_1(\epsilon)}{d\epsilon} = -a\left[b\left(\epsilon,\ \epsilon+p-\frac{1}{2}\right)f(\epsilon) - b\left(\epsilon+\frac{1}{2},\ \epsilon+p\right)f_1(\epsilon)\right], \tag{6}$$

where

$$j_1(\epsilon) = -b(\epsilon+½,\ \epsilon+½)\left(t\frac{df_1}{d\epsilon}+f_1(\epsilon)\right).$$

From the boundary conditions $j(0)=0$, $f_1(1/2)\approx 0$, $j(1/2)=j_1(0)$, and $f(1/2)=f_1(0)$ and from Eqs. (6) it follows directly that $j(\epsilon)+j_1(\epsilon)=j=\text{const}$, $j_1(0)=j$, $j_1(1/2)=0$, and $df_1/d\epsilon|_{\epsilon=1/2}\approx 0$. From an analysis of (6) at $\epsilon\sim 1/2$ we find that $f(1/2)=f_1(0)$, $df_1/d\epsilon|_{\epsilon=0}$, and j are exponentially small, so that $f(\epsilon)$ and $f_1(\epsilon)$ differ significantly from zero in the narrower interval $(0,\Delta)$, where Δ is estimated as the larger of the quantities t at $t^2/\alpha^2(p-1/2)^3$. Thus $f(\epsilon)$, which differs from zero at two "quasilevels," $\epsilon\approx 0$ and $\epsilon\approx 1/2$, acquires at $\omega=p+1/2$ a discrete character. The region of existence of the quasilevels is determined by the inequalities $1 \gg \gamma \gg \kappa\delta t/(p-1/2)^{3/2}$.

The system of quasilevels occurs at all $\omega=p+(n/m)$, where $n/m<1$ is the irreducible fraction. It includes m quasilevels $\epsilon_k=k/m$, $k=0,1,\ldots,m-1$, and for the number N_k of the electrons at these levels we have

$$\frac{N_0}{N_k} = \frac{p+\frac{2k+n}{m}-1}{p+\frac{n}{m}-1}\sqrt{\frac{p+\frac{k+n}{m}-1}{p+\frac{n}{m}-1}}.$$

The quasidiscrete spectrum $f(\epsilon)$ can appear only at $m \ll 1/t$. With increasing m, the average distribution of the electrons over the quasilevel approaches the distribution (5) at large $\alpha\omega'$. The appearance of the quasidiscrete spectrum $f(\epsilon)$ is illustrated in the figure.

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