Reflection of plane waves from an amplifying medium

A. A. Kolokolov

Moscow Physico-technical Institute
(Submitted April 28, 1975)
ZhETF Pis. Red. **21**, No. 11, 660–662 (June 5, 1975)

It is shown that the coefficient of reflection of light from an unbounded amplifying medium is a continuous function of the angle of incidence, and its modulus is always larger than unity.

PACS numbers: 42.10.F

Experimental[1,2] and theoretical[3,4] investigations of the reflection of light from an amplifying medium have shown that in the region of total reflection the modulus of the reflection coefficient can be larger than unity. Outside this region, the modulus of the reflection coefficients obtained in[3,4] is less than unity, and at the critical incidence angle the reflection coefficients are not uniquely defined and experience a discontinuity. These last results raise definite doubts, and we therefore consider this problem anew.

The determination of the reflection coefficients for an unbounded medium entails the choice of one of two possible refracted waves.[5] For a correct choice of the refracted wave, we consider first the reflection of a plane monochromatic wave from a plane-parallel amplifying layer of thickness l. The reflection coefficient for a wave polarized perpendicular to the plane of incidence is

$$\rho_\perp = \frac{\rho_{12} + \rho_{23}\exp(2i\Phi)}{1 + \rho_{12}\rho_{23}\exp(2i\Phi)}. \tag{1}$$

Here ρ_{12} and ρ_{23} are the usual reflection coefficients for the front and rear layers, and Φ is the phase shift of the refracted wave after one passage through the layer. In an amplifying medium, when the layer thickness increases, the value of $|\exp(2i\Phi)|$ increases without limit, so that expression (1) for an unbounded amplifying medium takes the form

$$\rho_\perp = \frac{1}{\rho_{12}} = \frac{k_{1z} + k_{2z}}{k_{1z} - k_{2z}}. \tag{2}$$

Here $k_{1z} > 0$ and $k_{2z} > 0$ are the components, normal to the layer, of the wave vectors of the incident and refracted waves, respectively. Formula (2) differs from the usual one in the sign of k_{2z}, a fact corresponding formally to a refracted wave propagating from the interface at an angle $\pi - \theta_2$, where θ_2 is the ordinary refraction angle. This result means that the Sommerfeld radiation principle cannot be used for an amplifying medium.

The reflection coefficient $\rho_\parallel$ for a wave polarized in the incidence plane is obtained from the usual formula by replacing θ_2 with $\pi - \theta_2$:

$$\rho_\parallel = \frac{n^2 k_{1z} + k_{2z}}{n^2 k_{1z} - k_{2z}}. \tag{3}$$

Here $n = n_2/n_1 = n_0(1 - i\kappa)$, and $\kappa > 0$ is the relative refractive index. We express the quantity k_{2z} in the form

$$k_{2z} = k_0 n_1 [n_0^2(1 - \kappa^2) - \sin^2\theta_1 - 2i\kappa n_0^2]^{1/2}, \tag{4}$$

where $k_0 = \omega/c$ and θ_1 is the incidence angle. The phase shift of the radicand lies in the interval $(0, -\pi)$ at all values of θ_1, and therefore $\mathrm{Re}\,k_{2z} > 0$ and it follows from (2) and (3) that the reflection coefficients are continuous functions of θ_1 at all incidence angles, and their moduli are always larger than unity. In the region of total reflection, expressions (2) and (3) coincide with those obtained in[2-4].

Substituting k_{2z} from (4) in (2), we easily find that $|\rho_{\perp}(\theta_1)|$ reaches its maximum value $|n_0 + 1/(n_0 - 1)|$ at $\theta_1 = 0$, and decreases monotonically with increasing θ_1. More interest attaches to the function $|\rho_{\parallel}(\theta_1)|$, which has one maximum at $\theta_1 = 0$ and a second maximum, much larger in magnitude, when θ_1 becomes equal to the Brewster angle θ_B:

$$\rho_{\parallel}(\theta_B) \approx \frac{2n_0^2}{\kappa|n_0^2 - 1|}, \qquad \kappa \ll 1. \tag{5}$$

Consequently, natural light incident at the Brewster angle on an amplifying medium becomes polarized in the incidence plane upon reflection. We note also that when the sign of κ is reversed, formulas (2) and (3) do not go over into the formulas that describe reflection from an absorbing medium.

Thus, the region of total reflection is not singled out from the point of view of the reflection coefficient. In this case, a special role is played by the Brewster angle, when it is possible to obtain large reflection coefficients at small values of κ.

The author is grateful to G. V. Skrotskiĭ for help with the work and S. A. Lebedev for useful discussions.

[1]B. Ya. Kogan, V. M. Volkov, and S. A. Lebedev, ZhETF Pis. Red. **16**, 144 (1972) [JETP Lett. **16**, 100 (1972)].
[2]S. A. Lebedev, V. M. Volkov, and B. Ya. Kogan, Opt. Spektr. **35**, 976 (1973).
[3]G. N. Romanov and S. S. Shakhidzhanov, ZhETF Pis. Red. **16**, 298 (1972) [JETP Lett. **16**, 210 (1972)].
[4]B. B. Boĭko, N. S. Petrov, and I. Z. Dzhilavdari, Zh. Prik. Spektr. **18**, 727 (1973); in: Kvantovaya elektronika i lazernaya spektroskopiya (Quantum Electronics and Laser Spectroscopy) Nauka i tekhnika, 1974, p. 449.
[5]L. I. Mandel'shtam, Lektsii po optike, teorii otnositel'nosti i kvantovoĭ mekhanike (Lectures on Optics, Relativity, and Quantum Mechanics), Nauka, 1972, p. 435.