Thermoelectric power in superconductors

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It is shown theoretically that in the presence of a temperature gradient there is produced in a superconductor an electric field that falls off exponentially with increasing distance from the superconductor boundary. The depth of penetration of the field greatly exceeds the correlation length and coincides with the length characterizing the "mixing of branches."

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Interest in the thermoelectric effect in superconductors,[1–3] first considered by Ginzburg,[4] has increased recently. It follows from the researches on isotropic superconductors that in the presence of a temperature gradient T'_x in a homogeneous and isotropic superconductor there is produced a phase gradient $2mv_s = \chi'_x$ of the order parameter, while the electric field E and the total current are equal to zero. However, this statement pertains to an unbounded superconductor. We shall show below that near the boundary of the superconductor there is produced a field $E(x)$ that falls off towards the interior of the superconductor.

We consider a semi-infinite superconductor occupying the half-space $x > 0$, in which a temperature gradient T'_x is produced. Near the critical temperature (this is the only case considered), the expression for the current is

$$j = \sigma E + \beta T'_x + j_s, \qquad j_s = eN_s v_s \quad , \tag{1}$$

where the coefficients σ and β are the same as in the normal metal. On the boundary of the superconductor (with a dielectric) one must satisfy the boundary condition

$$j_n(0) = (\sigma E + \beta T'_x)_{x=0} = j_s(0) = 0, \tag{2}$$

which is valid in the absence of surface recombination of the quasiparticles. Condition (2) cannot be satisfied if it is assumed that $E = 0$. Therefore, to solve the problem we need an equation that relates $E = -\phi'_x$ with j_s. Such an equation is one of the nonequilibrium Ginzburg–Landau equations obtained by Gor'kov and Éliashberg for gapless superconductors with paramagnetic impurities.

$$12\sigma|\Delta|^2\Phi - \xi^2(T)|\Delta_0|^2\frac{\partial j_s}{\partial x} = 0, \tag{3}$$

where $\xi(T)$ is the correlation length. The temperature gradient does not enter in this equation in the linear approximation, since the only quantity that could enter in (3), namely $\mathrm{div}(\vec{\nabla}T)$, is equal to zero by virtue of the conservation of the heat flux[1)] $\mathbf{q} = \kappa\vec{\nabla}T$. We substitute j_s from (1) in (3) and set $|\Delta|$ equal to the equilibrium value Δ_0 ($|\Delta|$ will vary in second order in T'_x). We obtain

$$\Phi''_{xx} - 12\xi^{-2}(T)\Phi = 0. \tag{4}$$

This equation describes the penetration of the field E into the superconductor. It was used to find this field in a superconductor with current flowing through the S–N boundary.[5,6] The solution (4) satisfying the conditions $j_n(0) = 0$ and $j(x) = 0$ then takes the form

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$$\Phi = -\frac{\beta \xi(T) T_x'}{\sigma\sqrt{12}} \exp(-\sqrt{12}x/\xi(T)). \quad (5)$$

In the considered case of momentum scattering by the impurities, the relation $\beta = -(\pi^2/3e)(T/\epsilon_F)\sigma$ holds true. Thus, the thermoelectric power produced at the boundary is the same as in a normal metal of thickness $\xi(T)/\sqrt{12}$.

We consider an ordinary superconductor with a gap. We linearize Eqs. (19) of[7], which describe the collisionless evolution of Δ in the BCS model, assuming the potential Φ in the Hamiltonian $\hat{H}$ to be given. We then obtain, for large time intervals, an analog of equation (3):

$$\frac{\partial}{\partial t}\left(e\Phi + \frac{1}{2}\frac{\partial X}{\partial t}\right) = \frac{v^2}{3}\frac{\partial^2 X}{\partial x^2}. \quad (6)$$

Equation (6) means that if there is a divergence of the superconducting current, then a gauge-invariant potential is produced, and in the BCS model this potential increases without limit. To obtain a finite value of the potential it is necessary to take inelastic scattering by phonons into account. This means that the characteristic time τ (and accordingly, the length l) within which the potential is established will greatly exceed Δ^{-1} [and respectively, $l \gg \xi(T)$]. We can therefore use the quasiclassical equations obtained by Aronov and Gurevich.[8] We linearize the kinetic equation for the deviation of the quasiparticle distribution function $\delta n = n - n_0(\tilde{\epsilon})$

$$v\frac{\xi}{\epsilon}\frac{\partial \delta n}{\partial x} - v\xi\frac{T_x'}{T}\frac{\partial n_0}{\partial \epsilon} = -\nu_- n_- - I_{\rm in}(n_+), \quad (7)$$

where $\delta n = n_+ + n_-$, $n_\pm$ are the symmetrical and asymmetrical parts of δn, $\xi = (p^2/2m) - \epsilon_F$, $\tilde{\epsilon} = [(\xi + e\Phi)^2 + |\Delta|^2]^{1/2} + pv_s$, ν_- is the frequency of the elastic collisions, and $I_{\rm in}$ is the integral of the inelastic collisions. To find Φ we use the Poisson equation

$$\frac{1}{4\pi}\frac{\partial^2\Phi}{\partial x^2} = e\delta N = e^2\frac{\partial N}{\partial \epsilon_F}\Phi + e\int dr_p n_+ \frac{\xi}{\epsilon}. \quad (8)$$

The coefficient of Φ on the right is the square of the reciprocal of the Thomas–Fermi screening length, so that the term on the left can be neglected. It is seen from (8) that we must calculate the integral of the quantity $\xi n_+/\epsilon$ with respect to the momenta. We therefore take the Laplace transform $n_+(s) = \int_0^\infty dx n_+(x)\exp(-sx)$, express n_+ with the aid of (7), and calculate the quantity

$$\bar{n}_+(s) = (2\pi)^{-3}\int \cdot dpp^2 d\phi n_+(s)\frac{\xi}{\epsilon} = -\frac{\beta\nu_-}{2ev^2}\frac{T_x' l}{(s+l^{-1})}, \quad (9)$$

where $l = v\sqrt{3\nu_-\nu_+}$, $\mu = p_x/p_0$, and the frequency ν_+ is defined by the equation $\int dpp^2 I_{\rm in}(n_+) = \nu_+ \int dpp^2(\xi/\epsilon)n_+$. Calculating the left-hand side of this equation, we obtain $\nu_+ = a(T^3/\theta_D^2)(\Delta/T)$, where $\alpha \sim 1$. The frequency ν_+ characterizes the so-called "mixing of branches," and is equal to the reciprocal time of equalization of the populations of the energy spectrum branches with $\xi > 0$ and $\xi < 0$. This frequency was introduced by Clarke and Tinkham.[9] To find $\bar{n}_+$, we used the condition $j_n(0) = 0$. We now substitute $\bar{n}_+$ from (9) in (8), take the inverse Laplace transform, and obtain $\Phi(x) = -(\beta/\sigma)T_x' \times \exp(-x/l)$. The quantity $\Phi(x)$ decreases exponentially at $x \gg l$. The characteristic fall-off length is equal to[2)] l. If we assume $\nu_- \approx 10^{10}$ sec^{-1}, $\nu_+ \approx 10^9(\Delta/T)$sec^{-1}, and $v \approx 10^8$ cm/sec, then $l \approx 3\times 10^{-2}(T/\Delta)^{1/2}$ cm. The predicted thermoelectric power $\Phi(0)$ can apparently be observed in experiment by measuring the temperature dependence of the thermoelectric power in an S—N circuit or in an intermediate-state superconductor, in analogy with the precedure used by Pippard and coworkers[10] to measure the excess resistance due to the penetration of the field E in the S region.

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[1)]The term γE in the heat flux can be neglected, just as in a normal metal, since it is small by virtue of the smallness of $(T/\epsilon_F)^2$.

[2)]The idea that the electric field in a superconductor should decrease over this length was advanced by Clarke and Tinkham.[9]

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In formula (7), ν should be taken to mean the x-component of the velocity $\nu_x = \mu\nu$; in the second line after Eq. (9), a factor ξ/ϵ was left out from under the integral sign in the left-hand side of the formula.

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