Two modes of collective excitations of a Fermi-liquid drop and sum rules for electromagnetic transitions in atomic nuclei

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(Submitted April 2, 1975)
ZhETF Pis. Red. **21**, No. 11, 670–673 (June 5, 1975)

The problem of the appearance of low-lying collective excitations in a drop of normal Fermi liquid is considered. In a quantum drop there are two collective-excitations modes, rather than one, the lower being the analog of ordinary capillary waves and the other the analog of zero sound. It is shown that both modes make comparable contributions to the sum rule.

PACS numbers: 21.60.E

It is well known that the problem of reconciling the experimental data with sum rules that pertain to electromagnetic transitions in atomic nuclei is the touchstone of the liquid-drop model. In the absence of forces that depend on the particle velocities (and these do not play an essential role in low-energy nuclear physics), the sum $S_L = \Sigma(E_S - E_0)Q_{0S}^2$, where $Q = r^L Y_{LM}(n)$, which reduces to the double commutator $S_L = \langle 0|Q[H,Q]|0\rangle$, can be easily calculated[1]:

$$S_L = \left(\frac{3Ze}{4\pi}\right)^2 \frac{R^{2L-2}}{2B_L^h}.$$

where $B_L^h \equiv B_1/L$ is the hydrodynamic mass coefficient. In the normalization that will be assumed below, it has the dimension of mass, $B_1 = (3/4\pi)M_0 A$. In classical hydrodynamics, the sum S_L is accounted for by a single transition $S_L^{(1)}$ from the ground state into the one-phonon state, for in this model we have $Q = (3Ze/4\pi)R^{L-1}\alpha$ (α is the phonon production operator), and

$$Q_{0,s}^2 = \frac{R^{2L-2}}{2\omega_L B_L^R}\left(\frac{3Ze}{4\pi}\right)^2.$$

However, the value of the mass coefficient B_L, determined from the experimental data on the frequencies and probabilities of the lowest collective excitations of the atomic nucleus, turns out as a rule to be much larger than the hydrodynamic value. The reason for this contradiction is that the atomic nucleus is a drop not of a quantum liquid but of a classical liquid, and in a drop of normal Fermi liquid there are two modes of collective excitations of different physical nature. One of them is the analog of ordinary capillary waves. Its appearance is due to spontaneous violation of translational invariance in the ground state of the system.[2] In this system, by virtue of the general theorems,[3] there always exists a collective-excitation mode that begins with $\omega = 0$.

A zero frequency is possessed by a dipole phonon with $L = 1$. The frequencies of the remaining excitations with $L \neq 1$ are positive. The origin of the low-lying surface branch can be understood also without the help of mathematical theorems. Let us cut out, say in the northern hemisphere of the drop, a narrow "crescent" of liquid and transfer it to the southern hemisphere in such a way that no changes, apart from the shift of the center of gravity of the drop, take place. Obviously, in this case the internal energy of the drop remains unchanged—the dipole rigidity is $C_1 = 0$ and consequently also $\omega_1 = 0$. We consider now not a dipole but an arbitrary deformation of a drop with small L. If the system is so arranged that an external field $V^0(r)$ applied at the point r changes the density ρ of the system only at that point, then the particles will not "feel" the difference between the dipole deformation or any other deformation of the surface, meaning that the internal energy remains unchanged (to distinguish dipole deformation from, say, quadrupole deformation it is necessary to compare the shift of the surface at a minimum of two surface points). Thus, in a system where $\delta\rho(r) \sim V^0(r)$, all the rigidities C_L will be equal to the dipole rigidity C_1, and consequently all the frequencies $\omega_L \equiv 0$. We note that all the transition densities—the form factors $\nu_L(r)$ of collective states—will also coincide with the dipole form factor $\nu_1(r) \equiv \partial\rho/\partial r$.[2] (This, incidentally, is indeed the hydrodynamic form factor.) Of course, the relation $\delta\rho(r) \sim V^0(r)$ is idealized. Even in the classical theory, where the quasiparticle mean-free-path is small, the change of $\rho(r)$ is determined by the value of the field $V^0(r')$ in the vicinity of $|r - r'| \sim r_0$ (r_0 is the average distance between the particles). Therefore the pure shift and deformation of a surface with $L \neq 1$ become nonequivalent, rigidity appears, and the degeneracy is lifted. It is easy to trace this fact directly in the equations, by writing down, for example, the equation for the form factor ν[4]

$$\nu_L(r,\omega_S) = \iint A_L(r,r',\omega_s)\mathcal{F}_L(r',r'')V_L(r'',\omega_0)\,d\tau'\,d\tau''. \quad (1)$$

Here $d\tau = r^2 dr$, $A_L(r,r')$ is the standard particle-hole propagator

$$A_L(r,r'\omega) = \iiint G(\mathbf{r},\mathbf{r},\epsilon+\frac{\omega}{2})\,G(\mathbf{r},\mathbf{r};\epsilon-\frac{\omega}{2}) \times P_L(\mathbf{nn}')\frac{d\epsilon}{2\pi i}\,\frac{dn\,dn'}{4\pi}. \quad (2)$$

and $\mathcal{F}$ is the local interaction of the quasiparticles.

At $L = 1$, Eq. (1) has a solution[2] $\omega_1 = 0$, $\nu_1(r) \equiv \partial\rho/\partial t$. The liquid drop is homogeneous and therefore a surface solution of the type $\partial\rho/\partial r$ is possible only if the interaction inside is $\mathcal{F}^{\text{in}} > 0$ (on the outside, the sign of $\mathcal{F}$ is always negative, corresponding to attraction). In the positive case ($\mathcal{F}^{\text{in}} < 0$), the solutions $\nu_1 \equiv \partial\rho/\partial r$ of (1) will be of the volume type, i.e., they will describe not a drop but certain other systems. We can state the following: the gas from which the drop is formed, with attraction between particles, is compressed until the local interaction $\mathcal{J}^{\text{in}}$ becomes positive. If the functions $A(\mathbf{r}, \mathbf{r}')$ and $\mathcal{J}(\mathbf{r}, \mathbf{r}')$ were δ-functions, then we would have $\delta\rho(\mathbf{r}) \sim V^0(\mathbf{r})$, the integral (2) would not depend on L, all the frequencies ω_L would vanish identically, and $\nu_L(r) \equiv \nu_1(r) \equiv \partial\rho/\partial r$. This, of course, is not the actual case, since $A(\mathbf{r}, \mathbf{r}')$ is not a δ-function and $\omega_L \neq 0$ at $L \neq 1$. Actual calculations show that the frequency ω_L of these oscillations—quantum capillary waves at $L \sim 1$—lies below the corresponding single-particle frequencies ω_L^{sp}, and therefore oscillations with small L are not damped. [The lifting of the degeneracy depends on the behavior of $A(\mathbf{r}, \mathbf{r}')$ over distances $(\mathbf{r} - \mathbf{r}') \gg \mathbf{r}_0$, and is entirely different in the quantum and classical cases.] But the classical surface peak of width $\sim r_0$ in $\nu_L(r)$, which is due to the coherent contribution of all the particles, remains also in the quantum case.

In a quantum drop there is one more collective-excitation mode, which is not connected with the surface oscillations. It exists also in a secured surface of a system, particularly in a gas of interacting particles that are locked in a potential box. At $T = 0$, the quasiparticles closest to the Fermi surface have a practically infinite time τ. Giant resonances have frequencies $\omega \gtrsim \epsilon_F A^{-1/3}$, i.e., the relation $\omega\tau \gg 1$ is satisfied. These are thus zero-sound excitations. Giant resonances constitute the start of the zero-sound mode of collective excitations of a large system.

If the propagator $A(\mathbf{r}_1, \mathbf{r}_2)$ were local, the two modes would never be intermixed. However, A has also long-range components[4] and as a result the real observed excitations are mixed.

In particular, in the form factor $\nu_L(r)$ of the lowest excitation there appear, besides the surface peak, also volume components (i.e., the quantum liquid becomes compressible).[2] An analysis shows that the quantum motion is also solenoidal. This mode can be called quantum capillary waves or "capons." By virtue of the solenoidal character of the motion, the mass coefficient B_L for the "capon" is larger than the hydrodynamic one, and consequently the sum S_L is not restricted to a transition with excitation of a "capon". But at a given L there is one more collective (volume) oscillation, namely giant resonance. Its collectiveness is of the order of $A^{2/3}$ (the number of particles in a layer $\sim \epsilon_F A^{-1/3}$ near the Fermi surface), and consequently [inasmuch as $(E_5 - E_0) \sim \epsilon_F A^{-1/3}$], its contribution to the sum S_L is of the same order as the contribution of the lowest excitation. The contribution of this state to the sum rule can be estimated and one can use more rigorously the condition for the normalization of the phonon production amplitude[4]

$$\left(g_L \frac{dA_L}{d\omega} g_L\right) = -1 \qquad (3)$$

and the definition of the excitation probability $w = (V_0 A_L g_L)^2$.

Using the smoothness of $g_L(r)$, we obtain from (3) the estimate $\bar{g}^2 \sim \epsilon_F^2 A^{-5/3}\omega^{-1}$, $w \sim (\bar{g})^2(A^2R^{2L}/\epsilon_F^2)$, and $S_L^{(2)} \sim R^{2L}A^{1/3}$, i.e., $S_L^{(1)}$ and $S_L^{(2)}$ are the same order of magnitude. Thus, in a quantum drop there are two different excitation modes, the lower being the quantum analog of the hydrodynamic mode and the other being the zero-sound mode, and both make comparable contributions to the sum rule, i.e., a Fermi-liquid drop is, as it were, a hybrid of a classical drop and an interacting Fermi gas.

In conclusion, the author is deeply grateful to A. B. Migdal for valuable advice and interest, and also to S. T. Belyaev, A. V. Ignatyuk, E. A. Sapershteĭn, and S. A. Fayans for fruitful discussions and critical remarks.

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