The PCAC condition in models with $(\bar{6}, 6)+(6, \bar{6})$ and $(8, 8)$ breaking of chiral $SU(3)\times SU(3)$ symmetry

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The consequences that ensue for $SU(2)\times SU(2)$ symmetry in models with $(\bar{6}, 6)+(6, \bar{6})$ and $(8, 8)$ breaking of chiral $SU(3)\times SU(3)$ are considered. It is shown that large deviations ($\sigma^{\pi\pi}\sim 1$ GeV) from $SU(2)\times SU(2)$ symmetry appear at $m_\pi^2=0$, meaning that it is impossible to introduce the PCAC condition in a consistent manner.

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At the present time the realization of chiral $SU(3)\times SU(3)$ symmetry is universally accepted with the following assumptions: (i) the symmetry breaking has a Goldstone character, (ii) the symmetry $SU(2)\times SU(2)$ is satisfied much less accurately than $SU(3)$, (iii) the PCAC condition can be introduced into the theory.[1] An important role in the statement of these principles was played by the model of Gell-Mann, Oakes, and Renner,[2] in which the symmetry-breaking part of the Hamiltonian (H_I) is transformed in accordance with the representation $(\bar{3},3)+(3,\bar{3})$ and assumptions (i)—(iii) are satisfied. σ is a commutator ($\sigma^{\alpha\beta}=i[Q_5^\alpha, \partial_\mu A_\mu^\beta]=[Q_5^\alpha[Q_5^\beta, H_I]]$) having only a component with zero isospin (here and below we have in mind the part symmetrical in the indices). Detailed calculations of the σ terms of πN and KN scattering[3] has shown that the $(\bar{3},3)+(3,\bar{3})$ model yields apparently values that are too low (26 and 180 MeV as against the experimental 50 and 450 MeV, respectively). The data on low-energy $\pi\pi$ scattering also call for the presence in the σ commutator of a term $I=2$.[8] Models were therefore proposed with H_I transforming in accordance with $(\bar{6},6)+(6,\bar{6})$ and $(8,8)$,[5,4] in which σ^{jk} ($j,k=1,2,3$) have components with $I=2$. We shall not consider here the experimental predictions of these models.[3] What matters to us is that in these models and at $m_\pi^2=0$ the axial current is not conserved, i.e., the $SU(2)\times SU(2)$ symmetry is not restored. Therefore an exceedingly important problem in these models is that of consistent introduction of the PCAC condition, which, as already mentioned, is supplementary in models of this type. Renner and Sudbery[6] have shown that under the conditions of octet dominance in the models $(\bar{3},3)+(3,\bar{3})$ and $(1,8)+(8,1)$ (trivially) the axial current is conserved in the limit as $m_\pi^2\to 0$ in operator form. In the models $(\bar{6},6)+(6,\bar{6})$ and $(8,8)$ no attempts were made whatever to establish the consequences of this phenomenon, for all its importance, although these models are widely used to extract concrete predictions.[3-5,7]

In this paper we consider the breaking of $SU(2)\times SU(2)$ symmetry in the models $(\bar{6},6)+(6,\bar{6})$ and $(8,8)$ at $m_\pi^2=0$. The Hamiltonian is written in the general case in the form

$$H=H_0+H_I=H_0+u_0+\epsilon u_8 \tag{1}$$

(we can also add the contribution of the 27-plet, but the accuracy with which the octet dominance is satisfied makes this unnecessary). Assumption (i) enables us, as usual, to express ϵ in terms of the masses of pseudoscalar mesons. We have

$$\epsilon_{6,6}=\frac{10\sqrt{5}(m_\pi^2-m_k^2)}{7(m_\pi^2+2m_k^2)}, \tag{2a}$$

$$\epsilon_{8,8}=\frac{10}{\sqrt{3}}\,\frac{2(m_\pi^2-m_k^2)}{(m_\pi^2+2m_k^2)}. \tag{2b}$$

We consider the models in the limit $m_\pi^2=0$. Conservation of the axial current requires $\langle B_\alpha|\sigma^{jk}|B_\beta\rangle=0$. We introduce the form factors

$$\langle B_\alpha|u_0|B_\beta\rangle=U_0\delta_{\alpha\beta}, \tag{3a}$$

$$\langle B_\alpha|u_\gamma|B_\beta\rangle=i\phi f_{\alpha\gamma\beta}+\delta d_{\alpha\gamma\beta}, \tag{3b}$$

$$\langle B_\alpha|u_\theta|B_\beta\rangle=U_{27}\zeta_{\theta\alpha\beta}. \tag{3c}$$

Here $\theta=1,2,\ldots,27$, $\zeta_{\theta\alpha\beta}$ are Clebsch-Gordan coefficients for the 27-plet in the 8×8 expansion, and U_0 is the contribution made to the baryon-octet mass as a result of breaking of the chiral $SU(3)\times SU(3)$. It has a reasonable order of magnitude 0—200 MeV. One usually puts $U_0\approx 200$ MeV under the assumption that the chiral $SU(3)\times SU(3)$ is a good symmetry. $\phi=(M_N-M_\Xi)/\epsilon\sqrt{3}$, and $\delta=2(M_\Sigma-M_\Lambda)/\epsilon\sqrt{3}$. The constant U_{27} is unknown. For the σ commutator we have in the $(8,8)$ model

$$\begin{aligned}\langle B_\alpha|\sigma^{jk}|B_\beta\rangle &= 12(1+C)U_0\delta_{\alpha\beta}\delta_{jk}+\frac{6(1+3C)}{\sqrt{3}}\\ &\times\delta_{jk}(i\phi f_{\alpha 8\beta}+\delta d_{\alpha 8\beta})-4\left(1+2C\right)\left(\zeta_{\theta jk}-\frac{1}{3}\delta_{jk}\zeta_{\theta ii}\right)\zeta_{\theta\alpha\beta}U_{27}\\ &+\delta_{jk}U_{27}\left[\frac{5+19C}{3}\zeta_{\theta ii}\zeta_{\theta\alpha\beta}-(1-C)\zeta_{\theta\alpha\beta}\zeta_{\theta 88}\right].\end{aligned} \tag{4}$$

	$(\bar{6},6)+(6,\bar{6})$	$(8,8)$
$\sigma_{NN}^{\pi\pi}$	0 (by definition)	0 (by definition)
$\sigma_{\Xi\Xi}^{\pi\pi}$	180	450
$\sigma_{\Lambda\Lambda}^{\pi\pi}$	– 17	– 820
$\sigma_{\Sigma\Sigma}^{\pi\pi}$ ($I=0$)	600	– 41
$\sigma_{\Sigma\Sigma}^{\pi\pi}$ ($I=2$)	485	1290

Here $C=(\sqrt{3}/10)\epsilon_{8,8}$. The analogous expression in the $(\overline{6},6)+(6,\overline{6})$ model is too complicated to write out here. We see that in (4) there is a contribution of both isospin 0 and isospin 2. Also, in the (8,8) model at m_π^2 we have $C=1$ and U_0 makes no contribution to the σ term, i.e., this uncertainty has no effect on the result. Let us set, by way of example, $\sigma_{NN}^{\pi\pi}=0$. From this we determine U_{27}, namely $U_{27}(6,6)=-156$ MeV and $U_{27}(8,8)=322$ MeV.

Without performing the calculations, we can immediately indicate two simple relations

$$\sigma_{NN}^{\pi\pi}(6,6)=\sigma_{\Xi\Xi}^{\pi\pi}(6,6)-\frac{12(M_\Xi-M_N)}{25}, \tag{5}$$

$$\sigma_{NN}^{\pi\pi}(8,8)=\sigma_{\Xi\Xi}^{\pi\pi}(8,8)-\frac{6(M_\Xi-M_N)}{5}. \tag{6}$$

We see there fore that in the models $(\overline{6},6)+(6,\overline{6})$ and (8,8) the breaking of $SU(3)$ immediately involves the breaking of $SU(2)\times SU(2)$, and it even exceeds in order of magnitude the average octet mass difference. Using the obtained values of U_{27} we find that the σ terms are numerically equal at $m_\pi^2=0$.

Although in the models $(\overline{6},6)+(6,\overline{6})$ and (8,8) the impossibility of satisfying exactly the PCAC condition $\partial_\mu A_\mu^j=m_\pi^2 f_\pi \pi^j$ is obvious beforehand, this circumstance has been neglected, the breaking being regarded as inessential. The numbers in the table show that the $SU(2)\times SU(2)$ breaking that appears in these models is larger by one order of magnitude than the pion mass and is due to $SU(3)$ breaking. Within the framework of the presently universally accepted realization of chiral $SU(3)\times SU(3)$, these breakings are described by different mechanisms which are not related in any way, since the chiral symmetry is assumed to be realized by the Goldstone-Nambu method, while $SU(3)$ is realized by the Wigner-Weyl method. Thus, models of breaking of chiral $SU(3)\times SU(3)$ with H_I in the form $(\overline{6},6)+(6,\overline{6})$ and (8,8) contradict the main premises (i)—(iii) of the theory, and are consequently not acceptable within this framework. The use of additional experimental information will make it possible to show[8] that in the case of combined breaking of symmetry of the type $(\overline{3},3)+(3,\overline{3})+(8,8)$, the aforementioned properties also lead to considerable deviations from PCAC.

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