Resonances on gliding orbits in an electron liquid

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We show that surface waves can propagate in metals having a noncylindrical Fermi surface.

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In weak magnetic fields, the electromagnetic properties of metals at microwave frequencies are determined by the surface states of electrons that glide along the surface of the metal (see[1-3]). Sharp resonant peaks of electromagnetic absorption, and also surface waves due to transitions between levels of gliding electrons are possible, in accordance with the previously employed model of free electrons, only for certain particular shapes of the Fermi surface of the conduction electrons. In this communication we wish to point out that allowance for the actual interaction between the electrons of the metal changes the situation qualitatively and makes it unnecessary to connect the existence of resonances of absorption and surface waves with the need for the presence of cylindrical Fermi surfaces.

Our analysis is based on the premises of the theory of the degenerate electron liquid of a metal.[4] Our purpose being to illustrate the possibilities uncovered by allowance for the interelectron interaction, we focus our attention here on resonant excitations of the spin-wave type in a metal, for the theory of which we can confine ourselves to a single Landau parameter B_0. The quantum formulation of the kinetic equations of the electron liquid, which is needed for the corresponding theoretical analysis, is contained in[5].

To demonstrate the reason why resonances due to gliding electrons become possible in metals with noncylindrical Fermi surface, we write out here the dispersion equation of long-wave spin waves whose frequency ω is close to the extremal value of the frequency of the resonant transition $\omega_{mn}(p_z)=\omega_m(p_z)-\omega_n(p_z)$ of the surface electrons:

$$1=\frac{B_0}{1+B_0}N_{mn}\int\frac{dp_z}{v_x(p_z)}\frac{\omega_{mn}(p_z)}{\omega-\omega_{mn}(p_z)},$$

$$N_{mn}=\frac{\hbar}{\pi}\int_{-\infty}^{\infty}dq\,|<m|e^{iqy}|n>|^2\left[\frac{1}{4\pi}\int\frac{ds}{|\mathbf{v}|}\right]^{-1}, \tag{1}$$

where the matrix element is calculated with the aid of the wave functions of the surface states (cf. [3]). Here $|\mathbf{v}|$ is the velocity and ds is the element of the Fermi surface. Finally,

$$\omega_{mn}(p_z)=\omega_m(p_z)-\omega_n(p_z)=(m^{2/3}-n^{2/3})\omega(p_z)$$

and we have neglected completely the electron collisions. In the particular case of a cylindrical Fermi surface, when the transition frequency is independent of p_z, we have for the frequency of the surface spin resonance

$$\omega=\omega_{mn}\left\{1+\frac{2B_0N_{mn}p}{v_x(1+B_0)}\right\} \tag{2}$$

where p is the Fermi momentum. In the opposite case of not strongly elongated Fermi surfaces, the main contribution to the integral (1) is made by values of p_z close to the extremum of the transition frequency. For example, if such an extremum corresponds to the central section, then

$$\omega(p_z)=\omega(0)+\frac{1}{2}\omega''(0)p_z^2 .$$

For undampled solutions of Eq. (1) to exist, it is necessary to satisfy simultaneously two inequalities

$$\frac{\omega_{mn}(0)-\omega}{\omega''(0)}>0,\qquad \frac{\omega_{mn}(0)-\omega}{\omega_{mn}(0)}B_0<0.$$

Thus, if the resonant frequency is close to the maximum value of the transition frequency and exceeds it, then the constant B_0 should be positive. The solution of (1) for not strongly elongated Fermi surfaces takes the form

$$\omega\approx\omega(0)(m^{2/3}-n^{2/3})\left\{1-\frac{2\omega(0)}{\omega''(0)}\left[\frac{\pi B_0}{v_x(1+B_0)}N_{mn}\right]^2\right\}. \tag{3}$$

The frequency shift turns out to depend on B_0 not linearly but quadratically (cf. [4]).

According to[3], in the case of a model with a spherical Fermi surface the electron gas does not contain surface waves near the surface-transition frequencies. In our analysis, for such a model, with allowance for the finite value of the wave vector **k**, we obtain for the spectrum of the surface spin waves

$$\omega=\omega_{mn}(0)\left\{1+\frac{3\pi^2}{4}\left(\frac{B_0}{1+B_0}\right)^2+\frac{3}{4}\frac{k^2v^2}{\omega_{mn}^2(0)}\right\}, \tag{4}$$

where v is the electron velocity on the Fermi surface.

The dispersion equation (1) does not take into account the possibility of collisionless Landau damping when the frequency ω is at resonance with a transition frequency $\omega_{rs}(p_z)$ that differs from $\omega_{mn}(p_z)$. Bearing in mind the proximity of the frequency ω to the limiting value $\omega_{mn}(0)$, we can state that this damping is small at small values of m and n. Thus, for example, in the case of a spherical Fermi surface the Landau damping is negligibly small if the following inequality is not satisfied

$$\frac{r^{2/3}-s^{2/3}}{m^{2/3}-n^{2/3}}-1\ll\frac{\omega-\omega_{mn}(0)}{\omega_{mn}(0)}\ll 1.$$

Obviously, this inequality does not hold for values of m and n on the order of ten. The resonance must therefore be sharp, and the surface waves can propagate.

The concrete results described here for surface spin excitations of a normal metal have properties that are common to surface excitations of an electron liquid. We can therefore state that the interelectron interaction is one of the common causes of the existence of different types of surface and resonant peaks of the surface impedance of metals in the transition-frequency range between the surface levels of the gliding electrons.

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