Restriction on the perpendicular momenta in the production of vector particles in renormalizable Yang–Mills models

L. L. Frankfurt and V. E. Sherman

Leningrad Institute of Nuclear Physics, USSR Academy of Sciences
(Submitted May 12, 1975)
ZhETF Pis. Red. **21**, No. 12, 736–739 (June 20, 1975)

It is shown that the main contribution to the cross section for the production of vector particles in collisions of high-energy fermions is made in renormalizable models of the Yang–Mills type by perpendicular momenta of the same order as the mass.

PACS numbers: 11.10.J, 11.10.G

The necessary condition for reggeization of a vector particle in perturbation theory is that there be no increase of the significant perpendicular momenta with increasing energy. Otherwise, the leading asymptotic corrections to the amplitude would have a doubly-logarithmic character: $M^{(n)} \approx g^2 s(g^2 \ln^2 s)^n$ (s is the square of the energy of the colliding particles in their c. m. s. and g^2 is the coupling constant). According to an erroneous statement in[1], the asymptotic form in models of the Yang-Mills type is indeed doubly logarithmic, and in addition $\ln s$ is the result of integration with respect to the perpendicular momenta of the intermediate vector particles. We show in the present paper that in the collision of high-energy fermions, in any renormalizable model, the main contribution to the cross section for the production of a vector particle is made by perpendicular momenta on the order of the mass. This statement has a power-law accuracy in s. The proof is based only on renormalizability of the theory and on the characteristic properties of the Yang-Mills vertex. The argumentation can apparently be extended also to processes in which an arbitrary number of vector particles is produced.

We consider the process $a+b \to a' + b' + B$, where a, b, a', and b' are fermions and B is a vector meson. The notation is explained in Fig. 1:

$$q = L_2 - L_1,\; Q = p_1 - p_2, \quad P = p_1 + p_2,$$
$$\nu = kP, \quad \nu_1 = 2kL_2 \quad s = L_1 p.$$

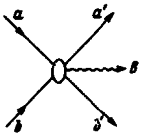

FIG. 1.

We are interested in the term of order s in the amplitude, and we wish to prove that the main contribution to the cross section comes from the region $q_\perp^2$, $Q_\perp^2 \sim 1$. (The independence of the dimensions is via the masses.) We note first that the region in which all the perpendicular momenta are large and are of the same order, $q_\perp^2 \sim Q_\perp^2 \gg 1$, makes a small contribution, owing to dimensionality considerations and owing to the renormalizability of the theory. Actually, in this region, the masses of all the particles can be neglected, and consequently the matrix element takes the form $A \approx s/Q_\perp^3$. Therefore the integral that arises in the cross section

$$d\sigma \sim \int d^2 q_\perp \, d^2 Q_\perp \frac{1}{Q_\perp^6} \tag{1}$$

converges rapidly, since the degree of the numerator is smaller than the degree of the denominator. The real "danger" lies thus only in the region where q^2 and Q^2 are significantly different.

Let us investigate, for the sake of argument, the case $q_\perp^2 \gg Q_\perp^2$. We consider first the multiregge-kinematics region from which the principal logarithmic contribution stems: 1, q^2, $Q^2 \ll \nu$, $\nu_1 \ll s$. The B meson can be emitted either from the ends of the fermion lines (Figs. 2a and 2b) or from the center of the diagram (Fig. 2c). Since each of the blocks Ba' and Bb' is asymptotic in the multiregge kinematics, the leading

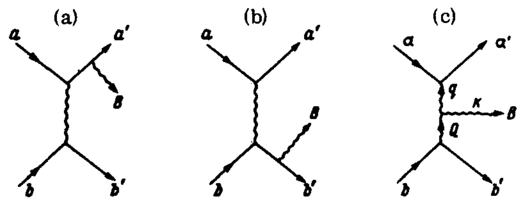

FIG. 2.

asymptotic contribution arises only when the wavy lines of Fig. 2 correspond to vector rather than scalar particles, and the vertex of the interaction of the three vector mesons (Fig. 2c) has a Yang-Mills character[2]:

$$\Gamma_{\tau\beta\alpha} \sim g_{\alpha\beta}(Q+q)_\tau - g_{\alpha\tau}(2q-Q)_\beta - g_{\beta\tau}(2Q-q)_\alpha. \qquad (2)$$

The proof is based on the following logic:

1. We shall show that in the given kinematic region the spin structure of all the significant diagrams, i. e., those slowly decreasing with increasing $q_\perp$, is factored out in the form of the multiplier $(\bar{u}\hat{\epsilon}u)$, where ϵ is the B-meson polarization vector while $\bar{u}$ and u are spinors corresponding to the fermions a and a'.

2. The contribution from the essential Feynmann diagrams, after multiplication by $\epsilon^L_\mu = k_\mu/M_B$, does not depend on q^2, since multiplication by k simply cancels out the adjacent propagators. The renormalizability of the theory, as is well known, denotes that the contribution from ϵ^L, which is independent of q^2 but is proportional to s, should cancel out in the sum of the diagrams.

3. The presence of factoring means that the cancellation takes place in the amplitude simultaneously in all polarizations. For the contribution of the transverse polarizations this cancellation leads directly to an additional factor $M_B/q_\perp$ in the amplitude ($M_B^2/q_\perp^2$ in the cross section), i. e., to a rapid convergence of the integral (1) with respect to $d^2q_\perp$. The contribution from ϵ^L, as is well known, is expressed in renormalizable models in terms of the amplitude of the emission of scalar "Goldstone" particles,[3] for which the region of large $q_\perp^2$ is inessential. In place of the third polarization $\tilde{\epsilon}_3 = \tilde{k}/M_B$ it is convenient to use $\tilde{\epsilon}_3 = \epsilon - \epsilon^L = (\tilde{k} - k)/M_B$, since the contribution of ϵ^L is small [here $\tilde{k} = (k_3, k_0, 0) \approx k + M_B^2/2k_0(-1, 1, 0)$. The third axis is directed along k]. Obviously, the ϵ_3 contribution is negligible at large $q_\perp^2$ since even multiplication of ϵ_3 by a momentum having large components does not lead to the appearance of $q_\perp$ in the numerator. By the same token we prove that the perpendicular momenta are bounded, with logarithmic degree of accuracy in s. For the statement to have power-law accuracy it is necessary to consider the kinematics. However, as will be seen from the proof, the same factorization for the transverse polarization takes place also in this region, a fact already noted in[4], so that the arguments remain in force.

We now prove the factorization properties. It is convenient to obtain the proof in the laboratory frame, where $p_\perp = (m, 0)$. We consider separately the contributions from the transverse polarizations $\epsilon^\perp_{1,2}$, which have no components that increase with energy, and from the polarization $\epsilon^L = k/M_B$. We note first that the emission of the B-meson from below (Fig. 2b) is insignificant in the region of large q^2, since the only contribution proportional to s comes from the polarization ϵ^L, but it decreases rapidly with increasing q^2. The structure of the fermion bracket for the diagram of Fig. 2a with emission of the B meson from above is the following:

$$\bar{u}\hat{\epsilon}(\hat{L}_1 + \hat{Q})\hat{P}u \approx s\,\bar{u}\hat{\epsilon}u. \qquad (3)$$

Equation (3) is satisfied for the polarizations $\epsilon_{1,2}$ in the region of quasi-two-particle kinematics, since the components of the vector Q are small in the laboratory frame at $Q^2 \ll q^2$, and the polarizations likewise do not contain large components. For the polarization ϵ^L, this is satisfied only in the multiregge region, for although the product (kQ) contains a term $\sim q^2$, it is nevertheless much smaller than ν_1.

A contribution to the quasi-two-particle kinematics appears in the diagram of Fig. 2c for the two polarizations $\epsilon_{1,2}$ only from the term $g_{\alpha\tau}$ in expression (2) for $\Gamma_{\tau\beta\alpha}$. This is a consequence of the fact that

$$(\epsilon^\perp q) = -(\epsilon^\perp Q), \text{ and } (\epsilon^\perp p_2) = 0 \qquad (4)$$

and all these products are small when $Q^2 \ll q^2$. For longitudinal polarization, the contributions from the terms $\sim g_{\alpha\beta}$ and $g_{\beta\tau}$ are significant, and cancel out in the multiregge region.

Indeed, in this region we have for longitudinal polarization

$$P_\beta L_{1\alpha} k_\tau [g_{\alpha\beta}(Q+q)_\tau - g_{\beta\tau}(2Q-q)_\alpha] \approx s[-q^2 + q^2] = 0. \qquad (5)$$

Only the term that increases with q^2 was calculated in (5), and the following relation was used:

$$\nu\nu_1 = -s q^2. \qquad (6)$$

Thus, for all the polarizations in the multiregge kinematics the contribution containing $q_\perp$ stems from the term $\sim g_{\alpha\tau}$ in the diagram of Fig. 2c,[2] and is proportional to $\bar{u}\hat{\epsilon}u$. This indeed proves the factorization.

The authors are grateful to V. N. Gribov for useful discussions.

[1] N. T. Nieh and Y. P. Yao, Phys. Rev. Lett. 32, 1074 (1974).
[2] C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
[3] G't Hooft, Nucl. Phys. B35, 167 (1967).
[4] L. L. Frankfurt and V. E. Sherman, Yad. Fiz. 19, 1338 (1974) [Sov. J. Nucl. Phys. 19, 684 (1974)].