Possibility of generating coherent optical radiation by an electron beam

N. E. Gertsenshteĭn and V. V. Kobzev

(Submitted April 21, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. 22, No. 1, 15–19 (July 5, 1975)

It is shown that higher harmonics are radiated when a "superluminal electron beam spot" made up of nonrelativistic electrons moves along a circle. The radiation power in the line is determined and the noise is estimated.

PACS numbers: 84.80.D, 42.80.

Coherent optical radiation can be obtained in resonant downward transitions of atoms and molecules (lasers) and in excitation of harmonics by passage of current through p-n junctions.[1] We show in the present paper that coherent optical radiation can be obtained also by a classical radio-engineering method, namely by exciting a resonator with an electron beam. We consider the concrete problem of frequency multiplication from the microwave to the optical band. We shall show that this multiplication can be obtained in principle in a single stage by exciting transition radiation in an optical resonator by means of a superluminal electron beam spot.[2,3]

We consider an electron beam passing through a circular-sweep system and focused on a flat metallic target (see the figure). We shall show that such a system radiates at the harmonics of the sweep frequency ω_0 and determine the radiated power. For simplicity we assume that the angle ψ is very small and therefore all the electrons travel parallel to the z axis. In this case

$$j_z \neq 0; \quad j_x = j_y = j_\perp = 0; \quad A = A_z; \tag{1}$$

and the equation of the electron trajectory is

$$x = \rho\cos\alpha; \quad y = \rho\sin\alpha. \quad \alpha = \alpha(t,\ z) = \omega_0\left(t - \frac{z}{v}\right); \tag{2}$$

The vector potential A of the electromagnetic field has only a z component. The boundary condition on an ideal conductor

$$E_{x,y}\Big|_{z=0} = 0; \quad \operatorname{div} A = \partial A_z/\partial z = 0 \tag{3}$$

can be replaced by an even continuation of A_z. We therefore have for A in the wave zone (Sec. 66 of[4])

$$A_z = 2\frac{e^{ikR}}{cR_0}\int j_\omega e^{-ik_\perp r_\perp}\cos k_\parallel z\,dv, \quad 0 \leqslant z < \infty. \tag{4}$$

Neglecting the discrete structure of the electron beam, the current has a discrete spectrum—lines at the frequency ω_0 and its harmonics

$$j_{n\omega_0} = \frac{1}{2\pi}\int_0^{2\pi} e^{in\omega_0 t} j(\mathbf{r},\ t)\,d\omega_0 t,$$
$$j_z = i_0\,\delta(x - \rho\cos\alpha)\,\delta(y - \rho\sin\alpha), \tag{5}$$

where i_0 is the beam current. We note the difference between the fields produced in circular motion of a real particle (Sec. 74;5 of[4]) and of an electron beam.[2,3]

$$\text{particle: } A_z = 0, \quad A_\perp \neq 0; \quad v = \omega_0\rho < c$$

$$\text{spot: } A_z \neq 0 \quad A_\perp = 0, \quad w = \omega_0\rho \gtrless c,$$

and it is this difference which leads to the difference between the formulas for the intensity. The calculations yield

$$A_z = 2i_0\frac{e^{ikR_0}}{cR_0}J_n\left(n\frac{w}{c}\cos\theta\right)e^{in\phi}\int_0^\infty e^{-in\frac{\omega_0}{v}z}\cos k_\parallel z\,dz. \tag{6}$$

For a nonrelativistic electron beam we have

$$v << c, \quad k_\parallel << n\frac{\omega_0}{v}; \quad \int_0^\infty e^{-in\frac{\omega_0}{v}z}\cos k_\parallel z\,dz = \frac{iv}{n\omega_0} = \frac{i}{k_n}\left(\frac{v}{c}\right); \quad k_n = n\frac{\omega_0}{c}; \tag{7}$$

meaning that the radiation is produced in a region located near the conducting surface at a distance on the order of $(1/k_n)(v/c)$. We are therefore dealing with transition radiation,[6–8] and the coherence is due to the sweep of the beam.

Since the region where the radiation produced is small, there is no need for slowing-down systems, which are extremely complicated for the optical and in-

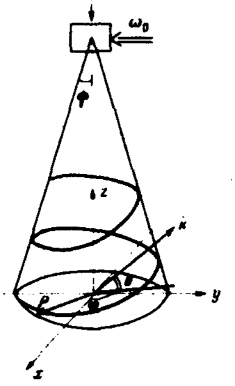

FIG. 1.

Copyright © 1975 American Institute of Physics

frared bands, owing to the miniature dimensions. For the radiation intensity we have the general formulas[4]

$$dI_n = \frac{c}{2\pi}\left|[k_n A]\right|^2 R_0^2\, do; \quad do = 2\pi\cos\theta\, d\theta, \quad 0 \leqslant \theta \leqslant \pi/2, \tag{8}$$

which yield

$$I_n = 4\frac{i_0^2}{c}\left(\frac{v}{c}\right)^2 S_n; \quad S_n = \int_0^{\pi/2} J_n^2\, n\left(n\frac{\omega}{c}\cos\theta\right)\cos^3\theta\, d\theta\ . \tag{9}$$

In the practical system of units we have

$$I_n = i_0^2 \frac{1}{\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{v}{c}\right)^2 S_n; \qquad \frac{1}{\pi}\sqrt{\frac{\mu_0}{\epsilon_0}} = 120\ \Omega.$$

For the beam spot we have $w \geqslant c$, but we confine ourselves to the case $w/c=1$, which is not optimal. Using the asymptotic expression in (Sec. 70 of[4])

$$n >> 1;\ \epsilon \sim 1;\ J_n(n\epsilon) = \frac{1}{\sqrt{\pi}}\left(\frac{2}{n}\right)^{1/3}\Phi\left[\left(\frac{n}{2}\right)^{2/3}(1-\epsilon^2)\right], \tag{10}$$

where Φ is an Airy function, we obtain

$$w = c; \quad S_n = \frac{2}{\pi n}\int_0^\infty \Phi^2(\xi^2)\, d\xi \sim \frac{1}{n}\ .$$

Just as in the case of a radiating electron,[4,5] the directivity pattern of the radiation tends to be compressed towards the conducting surface $z=0$. If resonance of the radiated field structure is attained with the aid of mirrors, then the line radiation power increases by a factor equal to the Q of the resonator.[9,10] We emphasize that the proportionality to Q can be demonstrated by using most general analyticity assumptions.[9] The formula (9) for I_n therefore acquires the factor Q/Q_0, where Q_0 pertains to ideally transparent mirrors, $Q_0 \sim k_n L$, and L is the distance between the trajectory of the beam spot and the mirror. The intensity of the harmonics decreases relatively slowly with increasing number, and therefore at $w/c=1$ the system under consideration can operate like a frequency multiplier with high multiplicity n. At a beam energy 60 kV, $n=1000$, and $Q/Q_0=10$, and at currents 1–100 μA, the power radiated amounts to 10^{-12}–10^{-8} W.

We consider now the natural line width in multiplication or, equivalently, the noise of the multiplier. We shall consider here only physical factors—the natural rather than the apparatus linewidth. The appearance of noise is due to the discrete—quantum—nature of both the electron beam and the electromagnetic field. The intensity of the transition radiation of an individual electron is[6]

$$w = \frac{4}{3}\frac{e^2}{c}\left(\frac{v}{c}\right)^2\frac{\Delta\omega}{\pi} = \frac{8}{3}\frac{e^2}{c}\left(\frac{v}{c}\right)^2 B, \qquad B = \Delta\omega/2\pi, \tag{11}$$

where B is the receiver bandwidth in Hz. When a follow-up filter or a automatic phase control system is used, the band B can be made very small. Each electron radiates a noise component independently of the others, so that we get for the noise power in the practical system of units

$$P_{\text{noise}} \approx \frac{2}{3}\left(\frac{v}{c}\right)^2 ieB\frac{1}{\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\ . \tag{12}$$

We note that the directivity patterns of the coherent and noise radiation are different, therefore the signal receiver will not receive an appreciable fraction of the noise. We neglect this circumstance. The signal/noise ratio is

$$\frac{I_n}{P_{\text{noise}}} \sim \frac{I}{6B}S_n\frac{i}{e}\ . \tag{13}$$

For a current 1.6 μA and $B=500$ Hz, the ratio $I_n/P_{\text{noise}} \sim 10^7$. The noise of a linear receiver for optical signals will be much larger

$$P_{\text{noise}} \sim \frac{\hbar\omega}{2}FB, \qquad F \geqslant 2, \tag{14}$$

where F is the noise factor. A power of 10^{-12} W corresponds to approximately 10^7 optical photons per second, and $I_n/P_{\text{noise}} \sim 2\times10^4$. Thus, the generated power is insufficient to measure frequency with high accuracy.[10] For formula (9) to be valid it is necessary to satisfy two conditions:

1) The *conductivity of the metal should be large* enough. This requirement only simplifies the derivation and is not obligatory in the general case.[7]

2) The dimension d of the electron beam spot should be small enough:

$$\mu = \frac{1}{2}k_n d << 1.$$

At $\mu \neq 0$, the line power decreases, and for a Gaussian spot shape we have

$$I \sim e^{-\mu^2/2}.$$

In long-focus systems, values $d\sim1$–$3\ \mu$ were obtained in practice at microampere currents,[11,12] and the spot dimensions were governed not by fundamental factors, nor by the space charge of the beam electrons, but by technical factors—the imperfection of the electron optics. At these spot dimensions, infrared radiation, with wavelengths longer than 5 μ, is possible, as in[1]. Short-focus systems of electron microscopes have spot dimensions on the order of 10–100 Å, but it is hardly possible to keep the spot small as it moves along a circle on the order of 1 cm diameter. If the spot dimensions can be decreased, then one can speak of generation in the visible and ultraviolet parts of the spectrum.

We have considered frequency multipliers whose operation, unlike self-excited generators, does not require the presence of feedback. This raises the natural question: Is it possible to construct a generator with self-excitation by using a resonator for a field structure typical of transition radiation? When a sufficiently intense electron beam passes through a localized electric field in the resonator, self-excitation is possible at definite values of the transit phases.[13] In this case there is no need to sweep the beam or to have small spot dimensions. An analysis of such a regime is beyond the scope of the present article.

The authors are grateful to S.A. Akhmanov and B.M. Bolotovskiĭ for valuable remarks.

[1]A. Javan, Appl. Phys. Lett. **15**, 398 (1969).
[2]B. M. Bolotovskiĭ and V. L. Ginzburg, Usp. Fiz. Nauk **106**, 577 (1972) [Sov. Phys.-Uspekhi **15**, 184 (1972)].
[3]B. M. Bolotovskii and V. L. Ginzburg, Éĭnshteĭnovskiĭ sbornik (Einstein Anthology), 1972.
[4]L. D. Landau and E. M. Lifshitz, Teoriya polya (Field theory), Nauka, 1967 [Addison-Wesley, 1971].
[5]D. D. Ivanenko and A. A. Sokolov, Klassicheskaya teoriya polya (Classical Field Theory), GITTL, 1949.
[6]V. L. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. **16**, 15 (1946).
[7]F. G. Bass and B. M. Yakovenko, Usp. Fiz. Nauk. **86**, 189 (1965) [Sov. Phys.-Uspekhi 420 (1965)].
[8]S. Mikhalyak, Candidate's dissertation, Moscow State Univ. 1961.
[9]A. I. Baz', Ya. B. Zel'dovich, and A. M. Perelomov, Rasseyanie, reaktsiya i raspady v nerelyativistskoĭ kvantovoĭ mekhanike (Scattering, Reaction, and Decays in Nonrelativistic Quantum Mechanics), Nauka, 1966.
[10]M. E. Gertsenshteĭn, A. N. Malakhov, and R. Ch. Bokun, Radiofizika **12**, 1401 (1969).
[11]K. Schlesinger, IRE Trans. ED-9, 281 (1962).
[12]M. Ardenne, J. of Sci. Instr. **34**, No. 5 (1957).
[13]F. B. Llewellyn-Jones, Electron Inertia (Russ. transl.), Gostekhizdat, 1946.