Useful energy from thermonuclear reactors

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The necessary conditions are indicated for obtaining useful energy from thermonuclear devices in which the ions are heated by energy transfer through Coulomb collisions with electrons. It is shown that this principle can be used to produce a reactor that employs the plasma pinch, described by the author in earlier papers, based of the reaction of deuterium with tritium. The time of the necessary plasma containment in pulsed installations with such an ion-heating method is estimated.

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It is well known that to obtain useful thermonuclear energy the ions in the plasma must have a high temperature. The main difficulty of ion heating is that the plasma is heated by an electric field, and in this case practically the entire energy is acquired by the electrons which, owing to the small mass, transfer the energy to the ions poorly in collisions. This transfer becomes even worse with increasing temperature. Therefore a more effective heating of the ions is realized in a plasma in certain collective processes,[4] but their use calls for a much more complicated reactor construction. The simple calculations presented here reveal, for both continuous[2] and pulsed reactors, the limits imposed on the effectiveness of thermonuclear processes when heat exchange between electrons and ions is effected by Coulomb collisions only.

The power transferred in a volume V from electrons with temperature T_e to ions with temperature T_i is given by (see, e.g.,[1] formula (4.20)):

$$P_a = V\frac{(T_e - T_i)kn}{t_{eq}} \text{ erg/sec}, \tag{1}$$

where k is Boltzmann's constant. The quantity t_{eq} determines the average time of redistribution of the energy by the collisions. It was obtained by Landau[6] (see also (5.31) of[3]) and is given by

$$t_{eq} = \frac{1}{6,7}\,\frac{m_0 k^{3/2}}{e^4\sqrt{m_e}}\,\frac{f}{\Lambda}\,\frac{T_e^{3/2}}{n} \text{ sec}, \quad m_0 > m_e\ , \tag{2}$$

where m_0 is the proton mass, $m_i = fm_0$ is the ion mass, m_e is the electron mass, e is the electron charge, and n is the plasma density. Since it is necessary to have $T_e > T_i$, we assume in our calculations

$$n = n_i = n_e = \frac{7,3\cdot 10^{21}}{T_e} P \text{ cm}^{-3} \tag{3}$$

P is the pressure in atmosphere and the logarithmic factor Λ is equal to:

$$\Lambda = \ln\left(\frac{3}{2\pi}\,\frac{k^{3/2}}{e^3}\,\frac{T_e^{3/2}}{n^{1/2}}\right) + \ln\left(\frac{4,2\cdot 10^5}{T_e}\right)^{1/2}, \quad T > 10^5\text{K}. \tag{4}$$

Substituting the values of all the quantities, we obtain for P_a:

$$P_a = V\cdot 5,5\cdot 10^{-19}\,\frac{\Lambda}{f}\,n^2\,\frac{T_e - T_i}{T_e^{3/2}} \text{ erg/sec}. \tag{5}$$

This power has a maximum. If it is assumed that the number of atoms in the volume V remains constant and the condition (3) is satisfied, then we obtain

$$P_a = P_{max}; \quad T_i = 0,6T_e \quad \begin{matrix} P = \text{const} \\ nV = \text{const} \end{matrix} \tag{6}$$

Consider the power transferred to the ions in the plasma pinch described in[1] and[2]. Its maximum value is

$$P_{max} = V\cdot 2,2\cdot 10^{-19}\,\frac{\Lambda}{f}\,\frac{n^2}{T_e^{1/2}} \text{erg/sec}. \tag{7}$$

Under continuous heating of the plasma, some of the supplied high-frequency power is also lost by radiation; this loss is (formula (5.60) of[3]):

$$P_r = V\cdot 1,57\cdot 10^{-27} n^2\sqrt{T_e} \text{ erg/sec}. \tag{8}$$

We assume that the double layer prevents heat from being transferred from the electrons to the surrounding

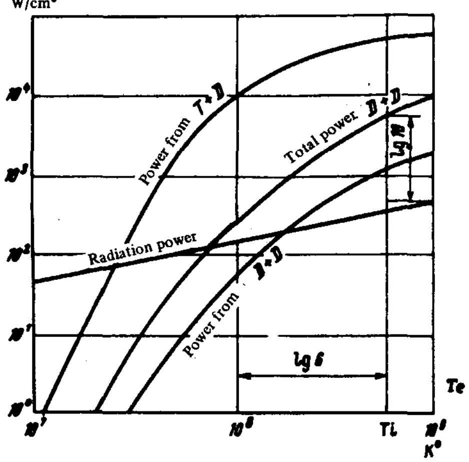

FIG. 1.

medium. Then, to obtain useful energy, the thermonuclear power P_t must exceed the lost power:

$$P_{max} + P_r = \eta P_t \quad , \tag{9}$$

where η is the efficiency of conversion of the nuclear power into the high frequency power delivered to the plasma. Once this coefficient is chosen, we have sufficient data for the determination of the temperatures T_e and T_i. Since the power P_t, like P_{max} and P_r, is proportional to the square of plasma density n, the temperatures T_e and T_i can be readily determined from the diagram taken from[5] and shown in the figure, where the ordinates represent the thermonuclear power and the abscissas the plasma temperature. The straight line determines the radiation power P_r. The necessary construction for the D + D reaction is given in the figure. We assume $T_i = 6\times10^8$ K and $T_e = 10^9$ K. From the diagram and from expression (9) we obtain $\eta = 0.24$.

The volume V of a plasma pinch of length $2l$ and radius b is

$$V = 2\pi l b^2. \tag{10}$$

Then the power P_{max} per unit pinch length at a pressure $P = 30$ atm, according to (3) and (7), is

$$\frac{P_{max}}{2l} = 2\ \text{kW/cm}; \quad \begin{matrix} P = 30\ \text{atm}; & \Lambda = 20; \quad T_e = 10^9; \quad f = 2 \\ b = 43\ \text{cm} & \eta = 2{,}2\cdot10^{14}\ \text{cm}^{-3}. \end{matrix} \tag{11}$$

This corresponds to the power determined by the ion thermal-conductivity loss assumed for the reactor described in[2]. The pressure is also equal to the assumed value. The obtained dimension of the plasma pinch lies within the limit assumed in[2]. Thus in the continuous process the plasma heating by Coulomb collisions, which is necessary in order to obtain energy, seems feasible. This heating, however, is less effective than the magnetoacoustic heating calculated in[4]. The reason is that at an electron temperature higher than that of the ions, there is not only an increase in the radiation losses, but also a decrease in the plasma density and a corresponding decrease in the thermonuclear energy yield. In addition, at electron temperatures as high as 10^9 K, in the presence of a strong magnetic field, losses to synchrotron radiation appear, and these losses can exceed the bremsstrahlung at low plasma densities.

If we use a mixture of deuterium with tritium in place of pure deuterium, then the efficiency of the thermonuclear reaction, as is well known, increases and the necessary plasma temperature decreases. The diagram shows the power from the D + T reaction. The necessary temperatures can be calculated in the same manner. They turn out to be $T_i = 7.8\times10^7$ and $T_e = 1.3\times10^8$. The efficiency of plasma heating increases by two orders of magnitude, and the necessary radius of the pinch increases by about 10 times.

The use of the D + T reaction makes the use of a reactor of small dimensions feasible. However, to decrease effectively the heat losses, a strong magnetic field is necessary, capable of twisting the ions onto orbits having a small enough radius in comparison with the cross section of the pinch discharge.

Expression (5) reveals also the restrictions that are imposed on the useful power in the case of pulsed thermonuclear reactions, if the ion heating in these is due to Coulomb heat transfer only. This can be simply done by assuming that the process begins with a rapid pulsed heating of the electrons to a certain temperature above T_e, and then, within a large time interval Δt, the ions are heated by the electrons to the temperature T_i at which the thermonuclear process takes place effectively.

Obviously, the time Δt must not be long enough to permit appreciable expansion of the plasma cluster.

Assume that the entire energy fed to the ions is due to Coulomb collisions. Then

$$\frac{P_a}{V} = kn\frac{\partial T_i}{\partial t} \quad , \tag{12}$$

From (5) we have

$$\Delta t = 2.5\cdot10^2\frac{f}{\Lambda}\int_0^{T_i}\frac{T_e^{3/2}}{n(T_e - T_i)}\,dT_i \ . \tag{13}$$

The ion heating is due to the thermal energy of the electrons, so that electron temperature will decrease. If we assume in the calculation that the electron temperature is constant at its final value T_e, and accordingly assume that the plasma density is also constant and is determined by the same temperature, then Δt is given by

$$\Delta t > -2.5\cdot10^2\frac{f}{\Lambda}\frac{T_e^{3/2}}{n}\ln\left(1 - \frac{T_i}{T_e}\right) \ , \quad n = \text{const}; \quad T_e = \text{const} \ . \tag{14}$$

The obtained quantity will here be smaller than the real one.

As an example of a pulsed thermonuclear process, we consider the tokomak. Modern designs of these reactors are described in[7]. All use the D + T reaction with a plasma at approximately $T_i = 5\times10^8$ K and $n = 3\times10^{13}$ cm^{-3}. The first to be heated are the plasma electrons. If the ion heating were due only to Coulomb collisions and if the electron temperature were $T_e = 10^9$, then according to (14) the heating time would be $\Delta t > 22$ sec. This is acceptable, since it appears that all the designs are based on the assumption that it is possible to contain plasma for more than 600 seconds. But to maintain the reaction for so long a time one uses in modern designs one of the collective methods of heating or direct introduction of heated ions. If the D+D reaction were to be used in the tokomaks, then according to expression (14) and the figure, the heating time would increase by two orders of magnitude. This is one of the main reasons why it is impossible to obtain useful power from the D+D reaction in a tokomak.

Similarly, a simple calculation illustrates why the thermonuclear reaction cannot be realized by Coulomb heating in a laser thermonuclear device. Here, too, the laser-radiation energy is initially received by the electrons. We consider the heating process for the D+T reaction in the same manner as for the tokomak. The only difference is that the initial density of the plasma cluster after the pulsed heating of the electrons will be determined by the density of the condensed state. We assume it to be

$$n = \frac{P}{m_0} = 4.9 \cdot 10^{22}\ \text{cm}^{-3}. \tag{16}$$

The temperatures T_e and T_i are assumed to be the same as in the preceding example. We then obtain from (14) for the ion-heating time

$$\Delta t > 3.9 \cdot 10^{-8}\ \text{sec}. \tag{17}$$

At a temperature $T_i = 5\times10^8$, the thermal velocity of the deuterium ion is $v_i = 1.8\times10^8$ cm/sec. It turns out that the radius over which the plasma expands during that time is

$$r = \frac{1}{2} v_i \Delta t > 3.5\ \text{cm}. \tag{18}$$

Let the initial plasma be a sphere of radius r_0, and assume that to heat it to a temperature T_e with a laser pulse we need j joules. In modern designs (see[7], p. 27), this energy is assumed to be $j = 10^6$ J. Then the initial radius of the sphere is $r_0 \approx 0.015$ cm and after a time Δt of ion heating the plasma density decreases because of expansion by a factor 10^7, so that it is impossible to heat the ions to the temperature required for the effective thermonuclear process. Of course, in the case of the D+D reaction, the ion heating will take even longer. As is well known, the laser thermonuclear device can be effective only if collective energy transfer from the electrons to the ions are used, and the use of cumulative compression is being planned for this purpose (see[7]).

The foregoing examples demonstrate the advantages of the continuous thermonuclear process, which can be effectively realized by Coulomb heating of ions in the reactor described in[2], even in the D+D reaction.

For a successful solution of the thermonuclear problem it is necessary to continue the experimental and theoretical study of the physical processes of heat transfer between the ions and the electrons in the plasma, and these processes serve as the basis for the feasibility of the effective production of thermonuclear energy.

[1] P. L. Kapitza, Zh. Eksp. Teor. Fiz. **57**, 1801 (1969) [Sov. Phys.-JETP **30**, 973 (1970)].
[2] P. L. Kapitza, Zh. Eksp. Teor. Fiz. **58**, 377 (1970) [Sov. Phys.-JETP **31**, 199 (1970)].
[3] L. Spitzer, Physics of Fully Ionized Gases, 2nd ed. 1965.
[4] P. L. Kapitza and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **67**, 1410 (1974) [Sov. Phys.-JETP **40**, 701 (1975)].
[5] W. E. Thompson, Nature **179**, 886 (1957).
[6] L. D. Landau, Zh. Eksp. Teor. Fiz. **7**, 203 (1937).
[7] F. L. Ribe, Rev. Mod. Phys. **47**, 7 (1975).

Erratum: Useful energy from thermonuclear reactors [JETP Lett. 22, No. 1, 10 (July 5, 1975)]

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The caption to the figure on page 10 is missing. It should read: Thermonuclear power at the density $n = 10^{16}$ cm^3.

On the same page, in formula (11), read "$n = 2.2 \times 10^{14}$ cm^{-3}" instead of "$\eta = 2.2 \times 10^{14}$ cm^{-3}."

