Crystallization of a two-dimensional electron gas in a magnetic field

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We predict crystallization of a two-dimensional electron gas in strong magnetic fields H in the density region where a Wigner crystal is impossible at $H = 0$. We construct the phase diagram of this system. We discuss the "cold" (at $T = 0$) melting of an electron crystal when its distance to the metallic substrate is changed. Conditions for the observation of these effects are indicated.

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The question of crystallization of a rarefied electron gas, which was posed by Wigner[1] approximately 30 years ago (see also[2–4]), has recently again attracted attention in connection with the possible formation of a Wigner crystal over the surface of liquid helium[5–7] and in inversion layers of semiconductors.[8] In this communication we predict crystallization of a two-dimensional system of electrons in strong magnetic fields H in the region of considerable surface concentrations of the electrons n_s, at which the Wigner crystal (in the absence of a magnetic field) is impossible. We discuss also the phase diagram of this system in terms of the variables r_0 and r_H (where $r_0 = (\pi n_s)^{-1/2}$ and $r_H = (\hbar c/eH)^{1/2}$) at $T = 0$. The localizing role of the strong magnetic field consists in an abrupt decrease of the amplitude of the zero-point oscillations of the electrons at

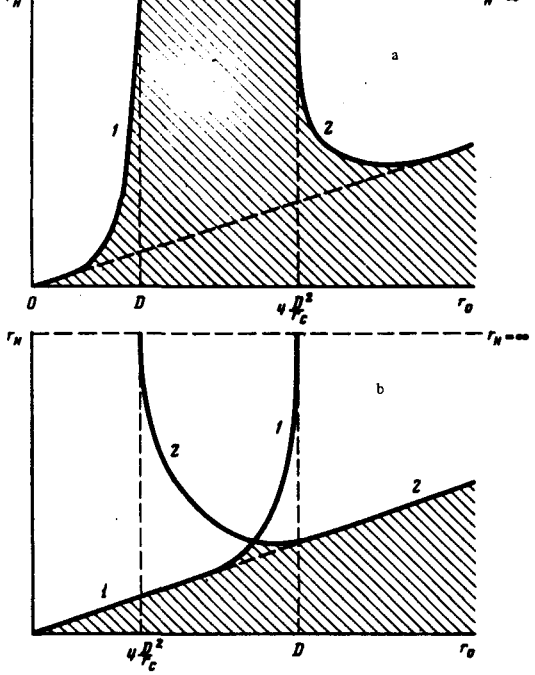

FIG. 1. Phase diagrams: a) $D > r_c/2$; b) $D < r_c/2$. The shaded area corresponds to the crystalline state. Curves: 1) $r_H = \gamma r_0[r_c/(r_c - r_0)]^{1/4}$; 2) $r_H = \gamma r_0[r_0/(r_0 - 4D^2/r_c)]^{1/4}$.

the lattice site, thereby ensuring its stability.[1)] A possible system for the observation of crystallization of electrons, in addition to electrons over helium and the inversion layer in semiconductors, may also be a charged ultraquantum film of a pure semiconductor, serving as one of the plates of a capacitor.[2)] These objects are constructed in accordance with the following scheme: the region $z < 0$ is a metal, and the regions $0 < z < D$ and $z > D + d$ are dielectrics with dielectric constants ϵ_1 and ϵ_2. The considered two-dimensional electron system corresponds to an ultraquantum ($r_0 \gg d$) charged layer of thickness d ($d \ll D$) between dielectrics. The potential of the interaction between the electrons in the layer, averaged over the wave functions of the transverse motion (to simplify the calculations we assumed that $\epsilon_1 \approx \epsilon_2 \approx \epsilon$) takes the form $V(\rho) = \tilde{e}^2[\rho^{-1} - (\rho^2 + 4D^2)^{-1/2}]$, where $\tilde{e}^2 = e^2/\epsilon$. The characteristic (Einstein) frequency ω_0 of the lattice oscillations at $H = 0$ is determined by the coefficient of the second term of the expansion of the lattice energy in terms of the displacement of one electron, with the remaining electrons fixed at the lattice sites. We obtain in this case for a quadratic lattice, $\omega_0 \approx 0.84\sqrt{\tilde{e}^2/m^* r_0^3}$ at $r_0 \lesssim 2D$ and $\omega_0 \approx 2.0\sqrt{\tilde{e}^2 D^2/m^* r_0^5}$ at $r_0 \gtrsim 2D$, where m^* is the effective mass of the electron in the layer. In a magnetic field, the characteristic frequency ω of the oscillations of the electron in the lattice site increases: $\omega = \sqrt{\omega_0^2 + \omega_H^2}$, where $\omega_H = eH/m^*c = \hbar^2/m^* r_H^2$.

The amplitude $u_0 = (\hbar/m^*\omega)^{1/2}$ of the zero-point oscillations decreases with increasing H. (A localizing action of the magnetic field had been noted—see e.g., the review[3]). According to the Lindemann Criterion, the crystal is stable at $u_0 \lesssim \gamma r_0$, where γ is an empirical quantity with a value 0.2 [illegible] tuting the quantities determined above into this inequality, we obtain the region of values of r_0 and r_H at which the electronic crystal exists. The results are shown in the form of phase diagrams in Figs. a and b for the cases $D > (1/2)r_c$ and $D < (1/2)r_c$ respectively; $r_c = a_0^*/\gamma^4$ is the smallest value of r_0 for which a Wigner crystal still exists at $H = 0$ and $D = \infty$ ($a^* = \hbar^2\epsilon/m^*e^2$). At $H = 0$, there is no electronic crystal not only at high densities but at low densities, namely, at $r_0 > 4D^2/r_c$. This is connected with the fact that interaction of the electrons at large distances is greatly weakened by the image forces, while the decrease of the interaction increases the amplitude of the zero-point oscillations $u_0 \sim r_0^{5/4}$, so that u_0/r_0 increases together with r_0 (rather than decrease as in a classical Wigner crystal with Coulomb interaction of the electrons). Thus, when the distance D to the metallic substrate is decreased, a "cold" ($T = 0$) melting of the electron crystal takes place. If $D < (1/2)r_c$, then crystallization at $H = 0$ is impossible for any density of the electron gas. For the system comprising an He film and a metal we have $(1/2)r_c \sim 300$ Å. For the inversion layer of a semiconductor we have $(1/2)r_c \sim 10^5$ Å. (We note that in typical experiments with inversion layers the distance D is of the same order of magnitude.) A magnetic field that crystallizes an electron gas of high ($r_0 \ll r_c$) and low ($r_0 \gg 2D$) density, for which the usual Wigner crystal does not exist at $H = 0$, is determined from the inequality $r_H \lesssim \gamma r_0$, that is to say, $H \gtrsim H_c = 10^{10}/r_0^2$ (gauss); r_0 is in angstroms. For $r_0 \sim 10^3$ Å ($n_s \sim 3 \times 10^{10}$ cm^{-2}), at which the Wigner crystal in the inversion layer or in a charged film of a semiconductor is impossible ($r_c \sim 10^5$ Å!), the field needed for crystallization is $H_c \sim 10^5$ G.

[1)] In strong magnetic fields the difference between the energies of the crystal lattice and the state with homogeneous electron density decreases and always becomes negative (the energy of the state with homogeneous electron density was calculated by us in the Hartree-Fock approximation in a quantizing magnetic field); thus, the crystalline state in strong magnetic fields becomes more favored energywise.

[2)] If the film is positively charged, then we are dealing with crystallization of not excess electrons but holes. The surface concentration of the excess carriers in the film is $n_s = \epsilon E/(4\pi de)$, and its maximum value is determined by the capacitor electric breakdown field $E = E_{cr}$.

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