Double stimulated Mandel'shtam-Brillouin scattering as a reason for reflection of radiation

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The results of the theory of double stimulated Mandel'shtam-Brillouin scattering in a plasma layer with a reflecting back boundary are presented. It is shown that the new phenomenon is an efficient mechanism for collimated reflection, in which the usual specular reflection is replaced by backward reflection.

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In this paper, we present the laws governing a new phenomenon, controlled by a new mechanism, which we call double stimulated Mandel'shtam-Brillouin scattering (SMBS), in which two pairs of electromagnetic waves (the incident and reflected waves and the scattered and Stokes components) interact with the same sound wave. This mechanism leads to increased backward reflection without any assumptions about the fixed spontaneous level of fluctuations, which corresponds to an absolute SMBS instability under oblique incidence of radiation on a plasma layer. The threshold for double SMBS turns out to be lower than the threshold for the usual convective SMBS instability. The mechanism proposed by us opens up a new path for understanding the observed characteristics of SMBS in a laser plasma.

In a planar uniform plasma layer with density n_e and thickness $l\,(0<x<l)$, we represent the high-frequency electric field $\mathscr{E}(\mathbf{r},t)$ and the low-frequency density perturbations $\delta n_e(\mathbf{r},t)$ in the form

$$\mathscr{E}_z(x,y,t)=\sum_{\sigma=\pm 1}[E_{0\sigma}(x,t)\exp(i\sigma k_0 x\cos\theta+ik_0 y\sin\theta-i\omega_0 t)$$

$$+E_{-1\sigma}(x,t)\exp(i\sigma k_0 x\cos\theta-ik_0 y\sin\theta-i(\omega_0-\omega)t)]+\text{c.c.};$$

$$\delta n_e(x,y,t)=-in_e\nu_1(x,t)\exp(2ik_0 y\sin\theta-i\omega t)+\text{c.c.}$$

The index $\sigma=1$ distinguishes waves traveling in the direction of incidence of the wave, $\sigma=-1$ distinguishes waves traveling in the direction of reflection from the plasma, $k_0=\frac{\omega_0}{c}\sqrt{1-n_e/n_c}$ is the wave vector of the pumping wave incident on the plasma at an angle θ, n_c is the critical density, $\omega_0=2k_0 v_s\sin\theta$ is the frequency of sound, and v_s is the velocity of sound. Our boundary conditions for the electromagnetic field are as follows: $E_{01}(0,t)=E_0$, $E_{-11}(0,t)=0$; $E_{\mu-1}(l,t)=E_{\mu 1}(l,t)re^{i\phi}$ $(\mu=0,-1)$, which differ from the usual boundary conditions[1] by the inclusion of partial specular reflection $(r\leqslant 1)$ at the back boundary of the layer. An abridged system of equations follows from the Maxwell's equations and the equations of acoustics with a ponderomotive force[2]:

$$\sigma\frac{\partial}{\partial x}E_{0\sigma}=-\frac{\alpha k_0}{2\cos\theta}E_{-1\sigma}\nu_1\,;\quad \sigma\frac{\partial}{\partial x}E_{-1\sigma}=\frac{\alpha k_0}{2\cos\theta}E_{0\sigma}\nu_1^*\,; \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\gamma_s\right)\nu_1=\frac{\omega}{32\pi n_c\kappa_B T}\left(E_{01}E^*_{-11}+E_{0-1}E^*_{-1-1}\right), \tag{2}$$

where $\alpha=n_e/(n_c-n_e)$, γ_s is the damping decrement of sound, T is the plasma temperature, and κ_B is Boltzmann's constant. Time derivatives are ignored in (1) because $v_s/c\ll 1$. The condition for the sufficiently high damping of sound, which enables the spatial derivatives in (2) to be ignored, will be discussed below.

The stationary problem ($\partial/\partial t=0$) of determining the coefficients of reflection in the backward $R_{-1}=E_{-1-1}(0)/E_0$ and in the specular $R_0=E_{0-1}(0)/E_0$ directions, where $|R_0|^2+|R_{-1}|^2=r^2$, is important for interpreting experiments on backward SMBS with oblique incidence of radiation. The system (1) and the boundary conditions reduce to an equation for $u(x)=(E_{01}E^*_{-11}+E_{0-1}E^*_{-1-1})/|E_0|^2$:

$$\frac{du}{dx} = \frac{\alpha k_0}{\cos\theta}\nu_1\left[\frac{1}{4}(1-r^2)^2 + |R_{-1}|^2 - |u|^2\right]^{1/2};$$

$$u(0) = R_0 R_{-1}^*,\ |u(l)|^2 = \frac{|R_{-1}|^2}{4r^2}(1+r^2)^2. \qquad (3)$$

The solution of the boundary value problem (3), supplemented by the stationary solution of Eq. (2), leads to the equation

$$\exp(2q\kappa) = \frac{2\sqrt{r^2 - |R_{-1}|^2}\,[2qr - (1-r^2)\sqrt{r^2 - |R_{-1}|^2}]}{(1+r^2)(2q - 1 + r^2 - 2|R_{-1}|^2)}, \qquad (4)$$

which determines the coefficient of reflection. Here

$$q = \left[\frac{1}{4}(1-r^2)^2 + |R_{-1}|^2\right]^{1/2}$$

is the usual coefficient of convective SMBS amplification, and $I = |E_0|^2/8\pi n_c \kappa_B T$. Figure 1 is a plot of $|R_{-1}|^2/r^2$ as a function of κ, i.e., as a function of the thickness of the layer and the angle and intensity of irradiation for three values of r^2 (1, 0.5, and 0.1). It follows from (4) that there is a threshold for backward stimulated scattering

$$\kappa = \alpha I \omega k_0 l / 8\gamma_s \cos\theta$$

reaching the minimum value $\kappa_{\min} = 1/2$ at $r = 1$. Near threshold $|R_{-1}|^2 \propto \kappa$. The restriction on backward reflection above threshold is manifested in the fact that $|R_{-1}|^2 \rightarrow r^2$ according to the law

$$I = |E_0|^2/8\pi n_c \kappa_B T.$$

Thus, Eq. (4) describes switching of specular oblique reflection at the fundamental frequency to backward reflection at a displaced frequency. In addition, the efficiency of this switching increases as the angle of incidence is increased.

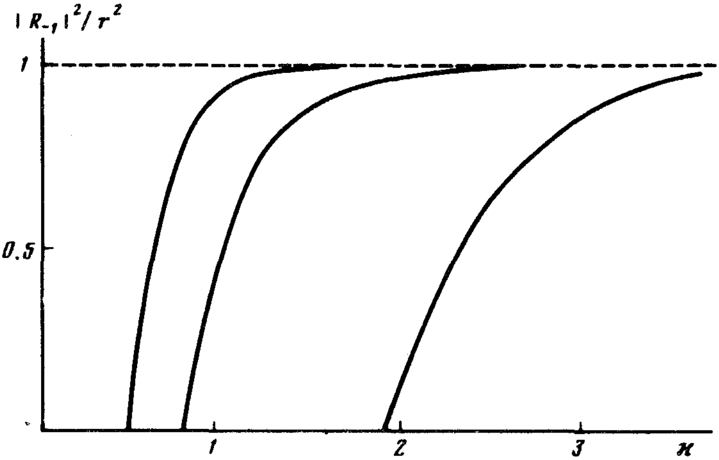

FIG. 1.

For sufficiently large values of ν_1, when $I > (\gamma_s/2\omega)^2$, Eq. (2) becomes inapplicable and the nonlinear approach to SMBS developed in Ref. 2 is required. Then, instead of (4), we obtain a system of two equations:

$$q\,\mathrm{cn}^2\,(\xi,\ 1/\sqrt{2}) = |R_{-1}|\sqrt{r^2 - |R_{-1}|^2}\ ;$$

and

$$q\,\mathrm{cn}^2(\xi - al,\ 1/\sqrt{2}) = \frac{|R_{-1}|}{2r}\,(1 + r^2),$$

where cn is the elliptic cosine and $a = (2\alpha k_0/3\pi\cos\theta)\sqrt{qI}$. Maximum backward reflection ($|R_{-1}| = r$) is reached for

$$\kappa_* = (3\sqrt{2}\pi\omega\,/\,64\gamma_s)\,\Gamma^2(1/4)\ .$$

Since κ_* is a factor of $\sqrt{I}(\omega/\gamma_s)$ greater than $\kappa_{\rm th}$, the coefficient of backward reflection does not reach its maximum value as rapidly due to the effect of the acoustical nonlinearity.

In the limit of high irradiation intensities ($I \gtrsim 0.1$), the quantity ν_1 is determined by the phenomenon of ion trapping,[3] when

$$\nu_1 \approx \nu_m = {}^1/_4\,(\sqrt{1+\Delta} - \sqrt{\Delta})^2,$$

where $\Delta = (v_s/v_{Ti})^2$, and v_{Ti} is the thermal velocity of ions. Then the boundary value problem (3) gives: $|R_{-1}|^2 = r^2\sin^2\beta l$, where $\beta\cos\theta = 2\alpha k_0\nu_m$. This value does not depend on the pumping intensity and is a limiting value. The picture obtained here corresponds to Sigel's ideas (see Ref. 4) on backward reflection from a diffraction sound grating with the difference that we have described reflection from a traveling sound wave.

The stimulated reflection discussed here, just as the usual reflection from a layer with transparent boundaries,[1] corresponds to absolute parametric instability for fixed, x-independent amplitudes of the incident and reflected pumping waves. The linear theory of instability ($\nu_1 \propto \exp\lambda\gamma_s t$) leads to the following dispersion equation:

$$\frac{1-r^2}{1+\lambda}\,\kappa = -2\pi n i + \ln\frac{1+r^2}{2r^2},\quad n \text{ is an integer}.$$

The nonlinear solution obtained above corresponds to generation of a mode with $n = 0$, when $\mathrm{Im}\lambda \neq 0$. The presence of higher-order modes ($n \neq 0$, $\mathrm{Im}\lambda \neq 0$) indicates the possibility of transient nonlinear states, but the threshold for their excitation is much higher ($\kappa_{\min,n} \cong 4\pi|n|$).

The phenomenon of double SMBS examined above is realized only for waves scattered close to the direction of pumping ($\Delta\theta \sim \kappa/k_0 l \ll 1$), and for this reason corresponds to collimated stimulated backward reflection. We shall point out the conditions under which the spatial derivatives ($\partial/\partial x$) in the equation for ν_1 can be ignored. When ion capture is important, this condition is easily realized: $\alpha\nu_m k_0 l_T \gg \cos\theta$, where l_T is the characteristic scale of variation of Δ. In application to Eq. (2), this condition has the form $8\gamma_s/\omega > (\alpha I/\sin\theta\cdot\cos\theta)^{2/3}$, which indicates that the strong dissipation approximation breaks down for small angles of incidence. We note that for $\theta\to 0$ the threshold for double SMBS increases $I_{\text{th}} \propto \theta^{-2}$. We emphasize here that double backward SMBS differs from the usual convective backward SMBS because of the dependence of the frequency ω on the angle θ. Finally, we note that in a spatially inhomogeneous plasma with a nonuniform flow of matter, the difference from the theory presented above reduces to the substitution

$$\kappa \to \frac{E_{\text{vac}}^2 \cos\theta_0}{64\pi n_c \kappa_B T} \frac{\omega\omega_0 L}{\gamma_s c} \ln(\omega_0 L/c)^{2/3},$$

where $\omega = 2\omega_0(v_s/c)\sin\theta_0$, θ_0 is the angle of incidence of the pumping wave on the plasma in a vacuum, and L is the scale of the density inhomogeneity at the turning point of the electromagnetic waves. Here the expansion of the plasma does not lead to a Doppler frequency shift, while the density inhomogeneity does not lead to dephasing of the ponderomotive force and the sound wave in regions with different density.

We have thus shown how the phenomenon of double SMBS changes the usual specular reflection without a frequency shift to backward reflection with a frequency shift.

[1]N. M. Kroll, J. Appl. Phys. **36**, 34 (1965).
[2]V. P. Silin and V. T. Tikhonchuk, Zh. Eksp. Teor. Fiz. **83**, 1332 (1982) [Sov. Phys. JETP **56**, 765 (1982)].
[3]W. L. Kruer, Phys. Fluids **23**, 1273 (1980).
[4]R. Sigel, J. Phys. (Paris) Coll. C6, Suppl. **38**, C6-35 (1977).

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