Hopping conductivity at low frequencies in a disordered metallic chain

O. N. Dorokhov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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The low-frequency region $0 < \omega < \omega_1(T)$, in which the one-dimensional (1D) conductivity is described by the law $\sigma(\omega) \propto \omega^{s(\omega,T)}$, is found for low temperatures $T \ll 1/\tau$ (τ is the free time of flight). The exponent $s(\omega,T)$ varies from zero for $\omega \to 0$ to one for $\omega \to \omega_1(T)$. At higher frequencies, $\omega > \omega_1(T)$, the 1D Austin-Mott conductivity $\sigma(\omega) \propto \omega T \ln^2(T/\omega)$ occurs.

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In a $1D$ disordered conductor, all electronic states are localized.[1,2] We are examining the case of weak localization, when the Fermi level $\epsilon_F \gg 1/\tau$, which corresponds to the experimental situation discussed below. It is assumed that the $1D$ nature of the electron motion is due to metallic chains with defects packed into a three-dimensional crystal. Since the dielectric constant is very high in such systems (of the order of several hundreds), the long-range Coulomb repulsion can be ignored. At low temperatures $0 < T \ll 1/\tau$ the dc conductivity is initiated by phonons. It is usually assumed [3–5] that the probability (per second) of a hop out of one localized state ϵ_ν into another ϵ_μ if $\delta_{\nu\mu} = \{|\epsilon_\nu - \epsilon_F| + |\epsilon_\mu - \epsilon_F| + |\epsilon_\nu - \epsilon_\mu|\}/2T \gg 1$, is

$$1/\tau_{\nu\mu} = \nu_{ph} e^{-\xi_{\nu\mu}}; \quad \xi_{\nu\mu} = \zeta_{\nu\mu} + \delta_{\nu\mu}, \tag{1}$$

where ν_{ph} is a phonon frequency, and $\zeta_{\nu\mu} = z_{\nu\mu}/l$ is the distance between states scaled to the localization length, which in $1D$ coincides (see Refs. 6–8) with the free path $l = v_F\tau$. Equation (1) takes into account only the exponential dependence on $\zeta_{\nu\mu}$ and $\delta_{\nu\mu}$. The electrical resistance between the nearest states ν, μ, which is proportional to $\tau_{\nu\mu}$ is $R_{\nu\mu} = R_0 e^{\xi_{\nu\mu}}$. Optimization of the hopping distance in (1) leads in the d-dimensional case to the Mott law [9,4,5] for the conductivity

$$\sigma(\omega = 0) = \sigma_0 \exp[-(T_0/T)^{1/(d+1)}]. \tag{2}$$

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However, such optimization is not applicable to an infinite chain ($d = 1$),[10–12] since the rare, high-resistance sections are the main contribution to the resistance.

Our purpose is to find the $1D$ low-temperature conductivity of a disordered metallic chain with exponential accuracy with respect to temperature. In the limit $\omega \to 0$, the conductivity must become a dependence of the type (2), taking into account Refs. 10–12, while at higher frequencies it must agree with the Austin-Mott equation[13,4,7,8]

$$\sigma(\omega) \propto \omega T [\ln(\nu_{ph}/\omega)]^{1+d}, \tag{3}$$

which corresponds to Debye losses due to hops within a pair of levels (pair approximation). We note that one-dimensional hopping conductivity is still studied in the literature (see Ref. 14) only for weakly or strongly compensated semiconductors, when in (1) $\xi_{\nu\mu} = \zeta_{\nu\mu} + \epsilon_3/T$ (R percolation). For this reason, our results are qualitatively different in nature.

It is shown in Refs. 6–8 that the states ν, μ are statistically independent and are distributed according to the Poisson distribution if $\zeta_{\nu\mu} > 2\ln(8/\tau|\epsilon_\nu - \epsilon_\mu|)$. Under these conditions it is possible, according to Ref. 10, to derive the probability distribution for the quantities $\xi_{\nu\mu}$, corresponding to neighboring states near the Fermi level

$$W(\xi) = \frac{2\xi}{\xi_0^2} \exp\left(-\frac{\xi^2}{\xi_0^2}\right), \tag{4}$$

where $\xi_0^2 = [\nu(E_F)Tl]^{-1} = \pi/(T\tau) \gg 1$ is the area per state in the space of the dimensionless variables ξ, δ. We note that in Ref. 12, for obscure physical reasons, a numerical factor $\Phi \sim 10$ was erroneously introduced into the exponent in (4).

In a dc current, an electron successively overcomes all of the smallest resistances $R_{\nu\mu}$, forming an infinite cluster. At $\omega \neq 0$, an electron moves in one direction for a finite time π/ω (half-period), and then moves in the opposite direction. Therefore, the chain contains "impassable" resistances (at a given frequency) for which $\tau_{\nu\mu} > \pi/\omega$, i.e., $\xi_{\nu\mu} > \xi(\omega)$, where

$$\xi(\omega) = \ln(\nu_{ph}/\omega). \tag{5}$$

These "impassable" resistances separate the chain into clusters that are accessible to electron motion. When $\xi(\omega)$ is less than the average distance between levels ξ_0, the pair approximation is valid. We are examining the lower frequency range

$$\omega < \omega_1(T) = \nu_{ph} \exp\left[-\left(\frac{\pi}{T\tau}\right)^{1/2}\right], \tag{6}$$

in which $\xi(\omega) > \xi_0$, and the clusters contain many states. The average length of a cluster is defined as

$$L(\omega) = l\xi_0 / \int_{\xi(\omega)}^{\infty} d\xi\, W(\xi) = l\xi_0 \exp\left(\frac{\xi^2(\omega)}{\xi_0^2}\right) \propto \omega^{-\left(\frac{T\tau}{\pi} \ln \frac{\nu_{ph}}{\omega}\right)} \tag{7}$$

and the average "passable" resistance as

$$< R >_{\omega} = \int_0^{\xi(\omega)} d\xi\, W(\xi) R_0 \exp(\xi) \Big/ \int_0^{\xi(\omega)} d\xi\, W(\xi). \tag{8}$$

For an infinite chain, the conductivity is determined from (8). With exponential accuracy, we find

$$\sigma(\omega) = \sigma_0 \Big/ \int_0^{\xi(\omega)} d\xi \exp\left(\xi - \xi^2/\xi_0^2\right). \tag{9}$$

Two characteristic regimes are possible in (9). When $\xi(\omega) < \xi_0^2/2$, i.e.

$$\omega > \omega_0(T) = \nu_{ph} \exp\left(-\frac{\pi}{2T\tau}\right), \tag{10}$$

the region near the upper limit makes the main contribution to the integral in (9), which leads to

$$\sigma(\omega) = (\omega/\nu_{ph})\sigma_0 \exp\left(\frac{\xi^2(\omega)}{\xi_0^2}\right) \propto \omega^{\left(1 - \frac{T\tau}{\pi} \ln \frac{\nu_{ph}}{\omega}\right)}. \tag{9'}$$

In the region $\xi(\omega) < \xi_0$, where the pair approximation is valid, the exponential in (9') should be ignored. Thus the frequency dependence matches the linear frequency dependence (3). [We recall that σ_0 in (9) is a nonexponential function of temperature.] On the other hand, for $\omega < \omega_0(T)$ ($\xi(\omega) > \xi_0^2/2$) conductivity (9) no longer depends on frequency and conforms to the activation law[10,11]

$$\sigma(\omega) = \sigma_0 \exp\left(-\frac{\pi}{4\tau T}\right). \tag{9''}$$

We shall briefly examine the conductivity of a chain of finite length L. As the frequency is reduced to

$$\omega(L) = \nu_{ph} \exp\left\{-\left[\frac{\pi}{T\tau} \ln\left(\frac{L}{l\xi_0}\right)\right]^{1/2}\right\} \tag{11}$$

the average length of a cluster $L(\omega)$ becomes equal to L. Therefore, results (9), (9'), and (9'') are valid for $\omega > \omega(L)$, while for $\omega < \omega(L)$ the conductivity of an average chain is constant and equal to $\sigma(\omega(L))$. For very long chains with $\omega(L) < \omega_0(T)$, this limitation is not important, since the activation law (9'') is already in force at frequencies $\omega \sim \omega_0(T)$. But, a different dependence arises if $\omega(L) > \omega_0(T)$:

$$\sigma(\omega < \omega(L)) = \left(\frac{L}{l\xi_0}\right)\sigma_0 \exp\left\{-\left[\frac{\pi}{T\tau} \ln\left(\frac{L}{l\xi_0}\right)\right]^{1/2}\right\}, \tag{12}$$

This dependence corresponds to Mott's law (2). We note that (12) yields the conductivity of the average chain. If we examine, in accordance with Ref. 10, a large collection of parallel noninteracting chains of finite length, then the conductivity will be deter-

mined by chains with resistances that are smaller than that of a mean-statistical chain. This leads (see Ref. 10) to an additional insignificant factor $1/\xi_0$ in the logarithm in (12).

In conclusion, we shall discuss the applicability of the $1D$ approximation. Let $1/\tau_\perp$ be the probability (per second) of a jump into a neighboring filament. The condition that the electron jumps more often along the chain than across the chain is $1/\tau_\perp \ll \omega_1(T)$. The frequency dependences (9) are valid if $\omega > 1/\tau_\perp$. The dc conductivity is (10) or (12) if $\omega_0(T)$ or $\omega(L)$ exceeds $1/\tau_\perp$. For the lowest temperatures when $1/\tau_\perp$, the conductivity must be three-dimensional (2) with $d = 3$. However, experiments with many quasi-one-dimensional compounds (see Refs. 12 and 15) prove that there exists a rather broad region from $T \sim 100$ K to $T \sim 15$ K, in which at $\omega = 0$ Mott's law (2) is satisfied with $d = 1$ and $T_0 \sim 0.5\text{–}1 \times 10^4$ K. This fact shows that $1/\tau_\perp$ can be ignored in this temperature range. Since $1/\tau \sim 200$ K, $l \sim 10^{-6}$ cm in these compounds (see Ref. 16), we obtain the reasonable estimate $\sim 9\text{–}15$ for the logarithm in (12). Thus it is possible to estimate, though very roughly, the probability of a transverse hop $1/\tau_\perp \sim \nu_{ph} \exp(-20)$.

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