Singularities of the dielectric-constant tensor (DCT) of the electron liquid in metals

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The role of the Fermi-liquid interaction in the analytical properties of the electronic DCT of metals is determined.

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The Fermi-liquid interaction between electrons, which plays a fundamental role in the formation of the quasiparticle–conduction-electron spectrum in metals, is manifested primarily in the quantitative characteristics of metals. Very few phenomena in which the Fermi-liquid interaction plays a decisive role have been discovered.[1)] At $\omega\tau \ll 1$ (ω is the frequency and τ is the relaxation time of the electrons), the kinetic equation for the electron distribution function can be reformulated in such a way that Landau's matrix $f(\mathbf{p},\mathbf{p}')$ does not appear in the "answers" for the conductivities.[3] At $\omega\tau \gg 1$, Landau's matrix cannot be eliminated, but the equations taking into account the Fermi-liquid interaction generally differ from the "gas" equations only numerically.

In this paper, we show that the Fermi-liquid interaction qualitatively changes the analytic properties of the DCT components in the collisionless limit ($kl \to \infty$, $l = v_F\tau$, v_F is the Fermi velocity of electrons, and k is the wave vector) at temperature $T = 0$.

In the gas approximation the DCT is

$$\epsilon_{\alpha\beta}^{(\mathbf{r})} = \delta_{\alpha\beta} + \frac{8\pi e^2}{(2\pi\hbar)^3\omega} \oint v_\alpha R v_\beta \frac{dS}{v} \equiv \delta_{\alpha\beta} + \frac{4\pi e^2}{\omega} < v_\alpha R v_\beta >, \tag{1}$$

$$\mathbf{v} = \partial\epsilon(\mathbf{p})/\partial\mathbf{p}, \qquad R = (\mathbf{kv} - \omega - i0)^{-1}.$$

The singularities of $\epsilon_{\alpha\beta}^{(\Gamma)}$ with respect to k are produced by the multiple zeros in the denominator R. The equations

$$w(x,y) \equiv \mathbf{kv} - \omega; \quad \partial w/\partial x = \partial w/\partial y = 0 \tag{2}$$

determine the point[2)] $\mathbf{p}_c(x = x_c, y = y_c)$ on the Fermi surface $\epsilon(\mathbf{p}) = \epsilon_F$, along which the integration in (1) is carried out, as well as the value $k = k_c$, for which the components of the DCT have singularities (ω and $\boldsymbol{\kappa} = \mathbf{k}/k$ are assumed to be given). If the Fermi surface is a sphere, then $k_c = \omega/v_F$ for any κ. In the case of anisotropic Fermi surfaces, there can be a spectrum of singularities (several k_c for one $\boldsymbol{\kappa}$), determined by the positive (for $\omega > 0$) extrema of $\boldsymbol{\kappa}\mathbf{v}$ as a function of x and y: $k_c = \omega/(\boldsymbol{\kappa}\mathbf{v})_{\text{extr}}$.

At $k = k_c$, the components of the DCT become infinite (if $v_\alpha^c v_\beta^c \neq 0$, $\mathbf{v}^c = \mathbf{v}(\mathbf{p}_c)$, or remain finite (if $v_\alpha^c v_\beta^c = 0$, then $d\epsilon_{\alpha\beta}^{(\Gamma)}/dk$ has singularities). If the Fermi surface is

spherical, the longitudinal component $\epsilon_{\parallel} = (\kappa\hat{\epsilon}\kappa) \propto \ln|\Delta k|, \Delta k = k - k_c$, while the transverse components $\propto \Delta k \ln|\Delta k|$. If the Fermi surface has an arbitrary shape, $\mathbf{v}_{\perp}^{c} \equiv \mathbf{v}_c - (\boldsymbol{\kappa}\mathbf{v}_c)\boldsymbol{\kappa} \neq 0$, and the transverse components of the DCT become infinite at $k = k_c$. Of course, we should keep in mind that the condition of electrical neutrality renormalizes the transverse part of DCT

$$\langle v_\alpha R v_\beta \rangle_\perp = \langle v_\alpha R v_\beta \rangle - \frac{\langle v_\alpha R(\vec{\kappa}\mathbf{v})\rangle\langle(\vec{\kappa}\mathbf{v})Rv_\beta\rangle}{\langle(\vec{\kappa}\mathbf{v})R(\vec{\kappa}\mathbf{v})\rangle}. \tag{3}$$

The singular parts of all components appearing in (3) become infinite in an identical manner

$$\langle v_\alpha R v_\beta \rangle \sim v_\alpha^c v_\beta^c \langle R \rangle, \quad \langle v_\alpha R(\vec{\kappa}\mathbf{v})\rangle \sim v_\alpha^c(\vec{\kappa}\mathbf{v}_c)\langle R\rangle,$$
$$\langle(\vec{\kappa}\mathbf{v})R(\vec{\kappa}\mathbf{v})\rangle \sim (\vec{\kappa}\mathbf{v}_c)^2\langle R\rangle,$$

as a result, the divergent terms in $\langle \mathbf{v}_\alpha R v_\beta \rangle_\perp$ cancel out. The nature of the divergence of $\langle R \rangle$ is determined by the local structure of the Fermi surface near the point $\mathbf{p}_c$. For a random orientation $\boldsymbol{\kappa}\langle R \rangle \propto \ln|\Delta k|$. In nearly all metals, however, there are directions in which the divergence becomes stronger ($\langle R \rangle \propto |k_c/\Delta k|^{\alpha}$, $0 < \alpha < 1$ and depends on the degree of flattening of the Fermi surface at the point ($\mathbf{p}_c$).

We shall show that due to the Fermi-liquid interaction the divergent terms are missing in all components of the DCT (the condition $kl \gg 1$ or $k \sim k_c = \omega/v_F$ means that $\omega\tau \gg 1$, i.e., it is necessary to take the Fermi-liquid interaction into account in this case). From the kinetic equation, taking into account the Fermi-liquid interaction, we obtain[4]

$$\epsilon_{\alpha\beta}^{(l)} = \delta_{\alpha\beta} + \frac{4\pi e^2}{\omega}\langle v_\alpha R(v_\beta - \omega J_\beta)\rangle, \tag{4}$$

while the vector $\mathbf{J} = \mathbf{J}(\mathbf{p})$ is the solution of the integral equation

$$\mathbf{J} + \langle \mathbf{k}\mathbf{v}')f(\mathbf{p},\mathbf{p}')R'\mathbf{J}'\rangle = \langle \mathbf{v}'f(\mathbf{p},\mathbf{p}')R'\rangle, \tag{5}$$
$$\mathbf{v}' = \mathbf{v}(\mathbf{p}'), \ldots .$$

Hence, $\mathbf{J}_c = \mathbf{v}_c/(\mathbf{k}_c\mathbf{v}_c)$. But, according to (2), $\mathbf{k}_c\mathbf{v}_c = \omega$, so that $v_\beta^c - \omega J_\beta^c = 0$, and the divergent term is missing in $\epsilon_{\alpha\beta}^{(l)}$.

Singling out the leading singularity, it can be shown that at $|\Delta k| \ll k_c$, in general

$$\epsilon_{\alpha\beta}^{(l)} = A_{\alpha\beta} + \frac{B_{\alpha\beta}}{\langle R\rangle}, \tag{6}$$

furthermore, $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are finite at $k = k_c$. We shall not write out their values. We note that $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are determined by all electrons on the Fermi surface, and not exclusively by the region around the point $\mathbf{p}_c$. If $B_{\alpha\beta} = 0$ (for example, for $\hat{\epsilon}_\perp$ in the case of a sphere), then the singularity in $\epsilon_{\alpha\beta}^{(l)}$ is weaker than in (6) (for a sphere $\propto \Delta k \ln|\Delta k|$. Because of the Fermi-liquid interaction, the singularity may not only be weakened but also intensified (in the case of an anisotropic Fermi surface, the singular part $\epsilon_{\perp}^{(l)} \propto \Delta k \ln|\Delta k|$ for $\mathbf{v}_{\perp}^{c} = 0$, while the singular part $\epsilon_{\perp}^{(l)} \propto \ln^{-1}|\Delta k|$).

Strictly speaking, Eq. (4) is valid for $\hbar k v_F \ll T$ and $\hbar\omega \ll T$. For $T \to 0$, the analysis cannot be restricted to the Boltzmann equation, and quantum analysis is required. We shall determine the changes to which this leads within the framework of the gas approximation (for the example of Lindhard's function for $\epsilon_{\|}$). In the case of a spherical Fermi surface, the logarithmic singularity at $k = k_c$ splits into two weaker singularities $\propto \Delta k_{\pm} \ln|\Delta k_{\pm}|$, $\Delta k_{\pm} = k - k_c^{\pm}$, $k_c^{\pm} = k_c(1 \pm \hbar\omega/4\epsilon_F)$. It is evident that the results obtained above are valid for $|\Delta k| \gg k_c \hbar\omega/\epsilon_F$, while the maximum possible value $\langle R \rangle \propto \ln(\epsilon_F/\hbar\omega)$ or $(\epsilon_F/\hbar\omega)^{\alpha}$.[3)] A quantum analysis may be necessary for a Fermi surface with a complicated shape if $\kappa \approx \kappa_c$ [for $\kappa = \kappa_c$, the "collar" $\kappa_c \mathbf{v} = 0$ has a point of self-crossing or a loop forms (vanishes) in it[6]] and $(\kappa v)_{\text{extr}} \ll v_F$, while $k_c \ll \omega/v_F$.

The singularities of the DCT examined above, as a rule, are not located on the "mass shell," determined by the dispersion equation. However, first, the "mass shell" itself (solution of the dispersion equation) depends considerably on the analytic properties of the DCT[4)] and, second, there are effects that depend directly on the nature of the singularities of the components of the DCT. We note three such effects.

a) Dynamic screening (high-frequency Friedel oscillations) is determined by the singularities of the components of DCT as a function of k for fixed κ and ω[8].

b) When an electromagnetic wave is excited in a metal, the dependence of the nonexponentially damped field (for $l \to \infty$) on the distance from the surface is determined by the singularities of the DCT and, in addition, the sharper the singularity, the weaker the damping.[9]

c) It is shown in Ref. 10 that the structure of the Fermi surface can be reconstructed from the frequency dependence of the effective cross section for scattering of light by electrons in the metal. The singularities of the DCT serve as reference points in the reconstruction procedure.

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[1)]These include spin waves in normal (nonferromagnetic) metals (Silin[1]).

[2)]If the Fermi surface has flat or cylindrical sections, then Eq. (2) can determine the line or section on the Fermi surface. We do not examine these cases.

[3)]If $T \gg \hbar\omega, \hbar/\tau$, then $\langle R \rangle \propto \ln(\epsilon_F/T)$ or $(\epsilon_F/T)^{\alpha}$.

[4)]The analytic properties of the Lindhard function were investigated comparatively recently.[7]

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