Cyclotron radiation from a dense plasma

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The cyclotron radiation of a hot plasma is determined by the relativistic tail on the electron energy distribution; in other words, this is essentially synchrotron radiation. Depression of the synchrotron radiation may substantially reduce the emission from a plasma with a sufficiently high density.

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1. Tsytovich[1] has shown that the magnetic bremsstrahlung of relativistic particles (synchrotron radiation) is greatly attenuated if the particles are moving through a dense plasma instead of a vacuum. This effect is well known in astrophysics, where it is called the "depression of synchrotron radiation" (see Ref. 2, for example). In the present letter we show that this depression effect may reduce the emission from a hot plasma ($T \gtrsim 25$ keV) substantially (by a factor of several units).

2. According to Kirchhoff's law, the flux density of the emission from a plasma, $(dI/d\omega)d\Omega$, differs only slightly from the equilibrium Rayleigh–Jeans flux density if the absorption coefficient for radiation incident on the plasma, $\eta(\omega,\Omega)$, is approximately unity. For a plasma with $L \gg 1$ m and $T \gg 25$ keV we have $\eta(\omega,\Omega) \approx 1$ at

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$\omega \lesssim \omega^* \sim 10\omega_e$ and $\eta(\omega,\Omega) \ll 1$ at $\omega \gtrsim \omega^*$, where ω^* is the so-called emission frequency. Correspondingly, the total emission from the plasma is given approximately by

$$I = \omega^{*3} T / 12\pi^2 c^2 \ .$$

Here we have taken into account the waves of one polarization (the extraordinary waves), which carry off most of the energy from the plasma. Trubnikov and Bazhanova[3] have shown that the accuracy of emission calculations is poor if the frequency ω^* is found from

$$2\kappa(\omega^*, \quad \theta = \pi/2) L = 1, \tag{1}$$

where κ is the spatial damping rate, and $\theta = \widehat{kB_0}$. For a plasma slab of thickness L we have $\eta = 1 - \exp(-2\kappa L)$.

Let us find $\kappa(\omega, \theta = \pi/2)$, taking depression effect into account. To calculate κ we work from the Einstein relation between the emissivity and absorptivity of a plasma, which can be written in the following form (see Ref. 4, for example) in the classical limit ($\hbar\omega \ll T$):

$$\kappa = -\frac{4\pi^3 c^2}{\omega^2} \int d\mathbf{p} \frac{dj}{d\omega d\Omega} \frac{df_0}{d\epsilon}, \tag{2}$$

where $(dj/d\omega)d\Omega$ is the emissivity of an individual electron.

The synchrotron radiation "forms" in a time $\Delta t \ll \omega_e^{-1}$, so it is conveniently calculated as a bremsstrahlung which arises in a "collision" of an electron with a magnetic field (see Refs. 5 and 6, for example):

$$\frac{dj}{d\omega\, d\Omega} = \frac{e^2 \omega\, \omega_e}{(2\pi)^3 c^2} |\mathbf{J}|^2,$$

$$\mathbf{J} = \int_{-\infty}^{\infty} dt\, \mathbf{v}(t) \exp(i\omega t - i\mathbf{kr}(t)). \tag{3}$$

Here the time is reckoned from the instant at which $\mathbf{v}(t) \| \mathbf{k}$.

For extraordinary waves propagating across a magnetic field we have

$$J \approx v_\perp \omega_e \int_{-\infty}^{\infty} dt\, t \exp(i\Phi(t)), \tag{4}$$

where

$$\Phi(t) = \omega t - \mathbf{kr}(t) \approx (\omega - kv_\perp)t + \frac{1}{3!} k\rho_e(\omega_e t)^3 - \frac{1}{5!} k\rho_e(\omega_e t)^5, \ \rho_e = \frac{p_\perp}{m_e \omega_{eo}}.$$

We assume that out of the entire Maxwellian velocity distribution the electrons which dominate the emission are those whose velocities are near the speed of light,

$$v \approx c\left(1 - \frac{1}{2}\left(\frac{m_e c}{p}\right)^2 + \frac{3}{8}\left(\frac{m_e c}{p}\right)^4\right),$$

and whose pitch angles $\chi = \mathrm{arctg} v_\perp / v_\parallel$ are approximately equal to $\theta = \pi/2$. We also assume that the plasma density is not too high ($\omega_{pe} \ll \omega$); under these conditions we have

$$\kappa \approx \frac{\omega}{c}\left(1 - \frac{1}{2}\left(\frac{\omega_{pe}}{\omega}\right)^2\right).$$

Under these assumptions we find

$$\Phi \approx \frac{1}{2}\tilde{\omega} t\left(\left(\frac{m_e c}{p}\right)^2 - \frac{3}{4}\left(\frac{m_e c}{p}\right)^4 + \left(\chi - \frac{\pi}{2}\right)^2 + \left(\frac{\omega_{pe}}{\omega}\right)^2\right)$$
$$+ \frac{1}{6}\omega\,\omega_{e0}^2\, t^3 \left(\frac{m_e c}{p}\right)^2\left(1 - \frac{3}{2}\left(\frac{m_e c}{p}\right)^2\right) - \frac{1}{5!}\omega\,\omega_{e0}^4\, t^5\left(\frac{m_e c}{p}\right)^4. \qquad (5)$$

Here we have used

$$\omega_e \approx \omega_{e0}\frac{m_e c}{p}\left(1 - \frac{1}{2}\left(\frac{m_e c}{p}\right)^2\right),$$

where $\omega_{e0} = eB_0/m_e c$. The integral over dt in (4) can be evaluated by the method of steepest descent;

$$J \approx i\frac{(2\pi)^{1/2} c}{\omega\,\omega_{e0}}\exp\left(-\frac{1}{3}\frac{\omega}{\omega_{e0}}\left(\frac{m_e c}{p}\right)^2\left(1 + \frac{3}{5}\left(\frac{m_e c}{p}\right)^2\right) - \frac{\omega}{2\omega_{e0}}\left(\chi - \frac{\pi}{2}\right)^2 - \frac{\omega_{pe}^2}{\omega\,\omega_{e0}}\right).$$

Substituting the latter expression into (3) and (2), and again using the method of steepest descent to integrate over $d\epsilon$ and $d\chi$, we find

$$\kappa \approx \frac{\pi^{1/2}\mu^{3/2}\omega_{pe}^2}{3c\,\omega_{e0}}\exp\left(-\left(\frac{9}{2}\frac{\omega}{\omega_{e0}}\mu^2\right)^{1/3} + \mu - \frac{1}{10}\left(\frac{9}{20}\mu^2\sqrt{\frac{\omega_{e0}}{\omega}}\right)^{2/3} - \frac{\omega_{pe}^2}{\omega\,\omega_{e0}}\right). \qquad (6)$$

Here $\mu = m_e c^2/T \gg 1$. The saddle point is at

$$p \approx \left(\frac{4}{3}\frac{\omega}{\omega_{e0}} m_e^2 c T\right)^{1/3} - \frac{1}{10} \times \left(\frac{3}{4}\frac{\omega_{e0}}{\omega T}\right)^{1/3} m_e^{4/3} c^{5/3}.$$

Expression (6) differs from that derived in Ref. 7 by the last term in the argument of the exponential function; this term reflects the depression effect. The correspondence between (6) and the result of Ref. 7 shows that our assumption of a governing role of fast electrons in the emission was correct.

3. Let us use (6) to determine the maximum emission frequency. In condition (1) we have the dimensionless quantities

$$\Lambda = \omega_{pe}^2 L/\omega_{e0} c,\ \mu = m_e c^2/T,\ q_e = (\omega_{pe}/\omega_{e0})^2,\ n = \omega/\omega_{e0}.$$

We write $n^* = \omega^*/\omega_{e0}$ as a function of $\beta_e BL$ (G·cm) $= (20/3)\Lambda T$ (keV), treating μ and

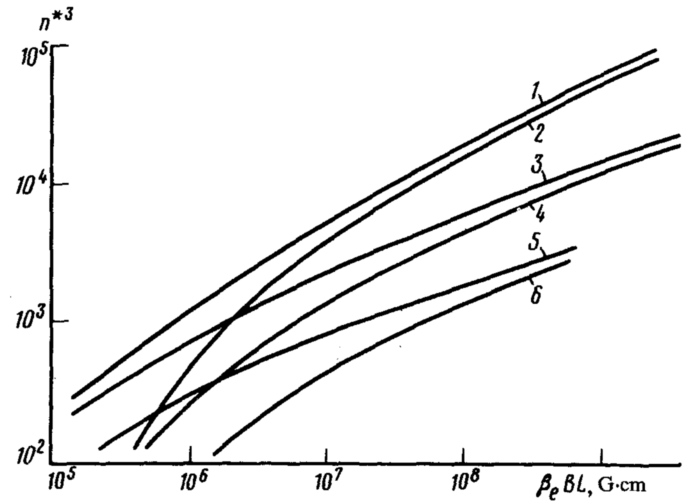

FIG. 1. Index of the maximum emitted harmonic vs $\beta_e BL$ for various plasma densities and temperatures. 1—T= 100 keV, q_e = 0; 2—T= 100 keV, q_e = 10; 3—50, 0; 4—50, 10; 5—25, 0; 6—25, 10.

q_e as parameters. With $q_e = 0$ we find the results of Refs. 3 and 7–9 from (1). (The error of these calculations does not exceed the error of the approximate equations derived in Ref. 9). With $q_e = 10$, the depression effect can reduce the emission by a factor of several units (Fig. 1). The effect becomes much greater as the plasma density increases (Fig. 2).

Figures 1 and 2 show that the depression effect can substantially reduce the cyclotron radiation at $q_e = \beta_e m_e c^2/2T_e \gtrsim 10$. Cyclotron radiation plays an important role in the energy balance of a hot plasma ($T_e \gtrsim 25$ keV). In this case the condition $q_e \gtrsim 10$ can be satisfied only at a sufficiently high plasma pressure, $\beta_e = 8\pi n_0 T_e/B^2 \gtrsim 1$. This relation among parameters is typical of so-called compact tori.

In several systems (bumpy tori, modified tandem mirrors, etc.) the plasma electrons consist of two groups: a main group with a temperature $T_e \ll m_e c^2$ and a small

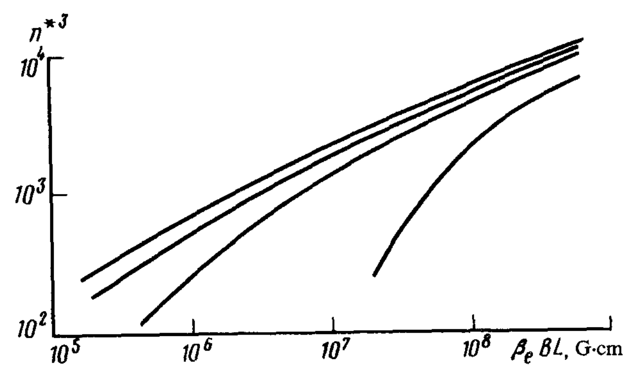

FIG. 2. The same as in Fig. 1, for T= 50 keV and various plasma densities. 1—q_e = 0; 2—3; 3—10; 4—30.

group of high-energy electrons with $\epsilon \gtrsim m_e c^2$. In systems of this sort the depression effect will substantially weaken the synchrotron radiation of the high-energy electrons at $\omega_{pe} \gtrsim \omega_{e0} \frac{\epsilon}{m_e} c^2$, where ω_{pe} is the plasma frequency of the "cold" electrons.[1,2,5] [This condition can easily be derived by analyzing the state of the phase resonance between the high-energy electrons and the electromagnetic waves; this state is characterized by the phase (5).] Under the experimental conditions of Ref. 10, for example, the value of q_e for the depression of the synchrotron radiation should be increased by a factor of about ten.

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