Langmuir solitons in an inhomogeneous unsteady plasma

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The excitation of sound by a Langmuir soliton which is being accelerated by a time-dependent potential force is analyzed for a plasma with a time-varying and inhomogeneous density distribution.

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One of the best-studied entities in nonlinear plasma dynamics is the soliton, which is of fundamental importance to our understanding of the physics of Langmuir turbulence with a high energy density[1] $W > (kv_T/\omega_p)^2 nT$, where ω_p and v_T are the plasma frequency and the electron thermal velocity, $n(x, t)$ is the density, $T = T_e + T_i$, and k is the spectral width. Chukbar and Yan'kov[2] and Kou *et al.*[3] have studied the conditions required for the existence of a soliton and the mechanism for the emission of ion acoustic waves by a Langmuir soliton whose acceleration results from an inhomogeneous density distribution in the plasma. In most of the corresponding experiments the plasma may be penetrated by modulated charged-particle beams or varying external fields, or it may be bombarded by electromagnetic waves. Small, low-frequency deviations $\eta(x, t)$ of the density from its equilibrium value n_0 give rise to a time-varying and inhomogeneous situation, in which the time variation of the plasma density and its inhomogeneity substantially alter both the law of motion of the soliton and the picture of the sound emission. Following Zakharov,[4] we describe these effects in terms of the temporal envelope (E) of the electric field of the high-frequency plasma waves (Langmuir waves),

$$E_L = \tfrac{1}{2}[E(x,t)\exp(-i\omega_p t) + \text{c.c.}],$$

and a low-frequency density variation δn, which obey the equations

$$2i\omega_p \partial E/\partial t + 3v_T^2 \partial^2 E/\partial x^2 = (\omega_p^2/n_0)(\delta n + \eta)E, \tag{1}$$

$$\left\{\frac{\partial^2}{\partial t^2} - n_0 v_s^2 \frac{\partial}{\partial x}\left[\frac{1}{n_0 + \eta + \delta n}\frac{\partial}{\partial x}\right]\right\}(\delta n + \eta) = \frac{1}{16\pi M n_0}\frac{\partial}{\partial x}(n_0 + \eta)\frac{\partial |E|^2}{\partial x}, \tag{2}$$

where v_s is the ion acoustic velocity. Under the assumption that (a) the velocity of the plasma drift caused by the inhomogeneity is small in comparison with v_s and in comparison with the soliton velocity, (b) the spatial and temporal inhomogeneity is slight, $\eta(x, t) \sim \delta n \ll n_0$, and (c) the derivatives of the inhomogeneity satisfy a corresponding condition, we can significantly simplify the original equations, which take the following form in terms of dimensionless variables:

$$2i\partial E/\partial t + \partial^2 E/\partial x^2 - (\eta + \delta n)E = 0, \tag{3}$$

$$\Box(t, x)(\eta + \delta n) = \partial^2 |E|^2/\partial x^2, \qquad \Box(t, x) \equiv \partial^2/\partial t^2 - \partial^2/\partial x^2. \tag{4}$$

Introducing the real self-similar amplitude $\mathscr{E}(x - \bar{x}(t))$ and the real phase $\varphi(x, t)$, we seek solutions of system (3), (4) in the form

$$E(x, t) = \mathscr{E}(x - \bar{x}(t))\exp i\varphi(x, t), \tag{5}$$

$$\delta n(x, t) = -\mathscr{E}^2(x - \bar{x}(t))/(1 - \dot{\bar{x}}^2) + N(x, t), \tag{6}$$

where $\dot{\bar{x}} = d\bar{x}(t)/dt$ is the velocity of the point around which the plasma waves are centered, and $N(x, t)$ is the perturbation of the density by ion sound outside the soliton. We assume $\dot{\bar{x}} \ll v_s \ddot{\bar{x}} \ll v_s/\Delta l$, so that we can ignore the inverse effect of the radiation field on the soliton and assume $N \ll \mathscr{E}^2$.

Substitution of (5) and (6) into system (3), (4) yields

$$\partial^2 \mathscr{E} / \partial x^2 - [2\dot{\varphi} + (\partial\varphi/\partial x)^2 + \eta(x,t)]\,\mathscr{E} - \delta n \mathscr{E} = 0\,, \qquad (7)$$

$$\partial \mathscr{E}^2/\partial t + \frac{\partial}{\partial x}(\mathscr{E}^2 \partial\varphi/\partial x) = 0, \qquad (8)$$

$$\Box(t,x) N(x,t) = -\ddot{\bar{x}}\,\partial\mathscr{E}^2/\partial x - \Box(t,x)\eta(x,t)\,. \qquad (9)$$

The last equation contains both the source of sound which is caused by the acceleration of the soliton and the sound which is caused by the external agent (the spatial inhomogeneity and the time variation).

Writing $\varphi(x,t) = x\dot{\bar{x}} + f(t)$, using the method of Ref. 3 to eliminate $f(t)$, and assuming that $\eta(n,t)$ is small over the width of the volume containing the plasma waves, we find the following equation for $\mathscr{E}$:

$$\frac{\partial^2 \mathscr{E}}{\partial x^2} - \Bigg\{ \mathscr{E}_0^2/2 + [2\ddot{\bar{x}} + \partial\eta/\partial x|_{\bar{x}}](x-\bar{x}) + 1/2\ \partial^2\eta/\partial x^2|_{\bar{x}}(x-\bar{x})^2$$

$$+ \mathscr{E}_0^{-2}\int_{-\infty}^{\bar{x}} dz\,[2\ddot{\bar{x}} + \partial\eta/\partial z|_{\bar{x}} + \partial^2\eta/\partial z^2|_{\bar{x}}(z-\bar{x})]\,\mathscr{E}^2(z,t)\Bigg\}\mathscr{E} + \mathscr{E}^3 = 0. \qquad (10)$$

Under the conditions

$$\mathscr{E}_0^2 \gg \partial^2\eta/\partial x^2|_{\bar{x}}(x-\bar{x})^2\,; \qquad 2\mathscr{E}_0^{-2}\partial^2\eta/\partial x^2|_{\bar{x}}\int_{-\infty}^{\bar{x}} dz\,(z-\bar{x})\,\mathscr{E}^2(z,t), \qquad (11)$$

this equation has a solution corresponding to a Langmuir soliton,

$$\mathscr{E}(x-\bar{x}(t)) + \mathscr{E}_0\,\mathrm{ch}^{-1}[(x-\bar{x}(t))/\Delta l]\,, \quad \Delta l = 2^{1/2}/\mathscr{E}_0\,, \qquad (12)$$

whose central point moves in accordance with

$$\ddot{\bar{x}}(t) = -1/2\ \partial\eta(x,t)/\partial x|_{\bar{x}}\,. \qquad (13)$$

In the time-varying case, the force exerted on the soliton depends explicitly on the time according to Eq. (13). This circumstance has fundamental consequences for the dynamics of the soliton and allows us to examine the transient and relaxation processes which occur when the external agent is turned on or off. A change also occurs in the density distribution in the ion sound which is radiated: $N(x,t) = N_1(x,t) + N_2(x,t)$, where $N_{1,2}$ correspond to the two different sources in Eq. (9).

Assuming homogeneous initial conditions on $N(x,t)$, and assuming the inequalities $\dot{\bar{x}} \ll 1$, $x\dddot{\bar{x}}\Delta l \ll \ddot{\bar{x}}$, we find a general expression for $N_1(x,t)$:

$$N_1(x,t) = 2^{-1/2}\mathscr{E}_0\Bigg\{\ddot{\bar{x}}(t-x+\bar{x}(t))\left[\mathrm{th}\frac{x-\bar{x}(t)}{\Delta l} - \mathrm{th}\frac{x-\bar{x}(0)-t}{\Delta l}\right]$$

$$+ \ddot{\bar{x}}(t+x-\bar{x}(t))\left[\mathrm{th}\frac{x-\bar{x}(t)}{\Delta l} - \mathrm{th}\frac{x-\bar{x}(0)+t}{\Delta l}\right]\Bigg\}. \qquad (14)$$

We must substitute the solution of Eq. (13) into Eq. (14) for various inhomogeneity profiles and time variations $\eta(x, t)$, while an explicit expression for $N_2(x, t)$ can be calculated for each $\eta(x, t)$ by using

$$N_2(x, t) = -\frac{1}{2}\int_0^t dt' \int_{x-t+t'}^{x+t-t'} dz\, \Box(t', z)\,\eta(z, t'). \qquad (15)$$

Let us consider some examples of explicitly time-dependent inhomogeneity profiles which are linear or quadratic in the coordinate x.

1) For a sinusoidal modulation of the inhomogeneity, $\eta(x, t) = 2\alpha x \cos\beta t\,\theta(t)$, the soliton moves along a trajectory $\bar{x}(t) = x_0 + V_0 t + (\alpha/\beta^2)(1 - \cos\beta t)$, where (here and below) $x_0 = \bar{x}(0), V_0 = \dot{\bar{x}}(0)$, and $N_2(x, t) = 2\alpha x(1 - \cos\beta t)$.

2) For the transition from a homogeneous plasma to an inhomogeneous one, with $\eta(x, t) = 2\alpha x(1 - e^{-\gamma t})$, the center of the soliton moves in accordance with

$$\bar{x}(t) = x_0 - (\alpha/\gamma^2)(1 - e^{-\gamma t}) + (V_0 + \alpha/\gamma)t - \alpha t^2/2.$$

It follows that at $t \gg \gamma^{-1}$ the motion is an "equivariable" motion. In this case we have $N_2(x, t) = 2\alpha x(\gamma t + e^{-\gamma t} - 1)$.

3) The transition from the inhomogeneous plasma to the homogeneous one is characterized by

$$\eta(x, t) = 2\alpha x e^{-\gamma t}\theta(t),$$

with

$$\bar{x}(t) = x_0 + (V_0 - \alpha/\gamma)t + (\alpha/\gamma^2)(1 - e^{-\gamma t});$$

at $t \gg \gamma^{-1}$, the soliton is in a steady state. The radiated sound field is

$$N_2(x, t) = -2\alpha x(\gamma t + e^{-\gamma t} - 1).$$

4) If the inhomogeneity has the quadratic profile

$$\eta(x,t) = -\alpha^2 x^2 \theta(t)(\sin^2\alpha t + \cos\alpha t),$$

the soliton oscillates with a period $T = 2\pi/\alpha$ between the values $x_0 \leqslant \bar{x}(t) \leqslant x_0 e^2$ in accordance with $\bar{x}(t) = x_0 \exp[2\sin^2(\alpha t/2)]$; in this case, $V_0 = 0$. We find

$$N_2(x, t) = \alpha^2 x^2[\cos\alpha t - (\cos 2\alpha t + 1)/2] + 2\alpha^3 t^3(\sin 2\alpha t - \sin\alpha t)$$

$$- \alpha^2 t^2(2\cos\alpha t - \cos 2\alpha t).$$

This analysis may be pertinent to the laser heating of targets and for estimating the degree of time variation and inhomogeneity of the plasma density in alternating external fields, by studying the dynamics of Langmuir solitons and the spectrum of the ion sound emitted by them.

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