Relationship between the components of the magnetoresistance tensor under conditions of the quantum Hall effect

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The linear relation between the deviation of the Hall resistance $\delta\rho_{xy}$ and the resistivity ρ_{xx} in the region of the wings of the plateau of the quantum Hall resistance in Si MIS structures, $|\delta\rho_{xy}| = (0.3\text{–}0.4)\rho_{xx}$, is observed experimentally. This relation indicates that the dominant reason for the increase in ρ_{xx} and $|\delta\rho_{xy}|$, in the region of the wings of the plateau, is the change in the density of mobile carriers, rather than the finite dissipation of energy. This result permits describing theoretically the phenomena observed in the experiment.

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All theoretical models proposed to explain the quantum Hall effect[1,2] explain equally well the results of experiments in the "zeroth approximation": quantization of the Hall component of the resistivity tensor

$$\rho_{xy} = h/Ne^2 \tag{1}$$

and vanishing of the diagonal component

$$\rho_{xx} \to 0 \quad \text{for} \quad T \to 0. \tag{2}$$

The individual differences between the theoretical models appear in the next approximation, for example, in estimating the magnitude of the possible corrections $\delta\rho_{xy}$ to the ideal value ρ_{xy} (1). For this reason, measurements of the corrections $\delta\rho_{xy}$ will permit choosing one theoretical model over the others.

It is therefore natural to seek the corrections $\delta\rho_{xy}$ in the form of an expansion in powers of the small parameter (ρ_{xx}/ρ_{xy}):

$$\delta\rho_{xy}/\rho_{xy} = \alpha_0 + \alpha_1(\rho_{xx}/\rho_{xy}) + \alpha_2(\rho_{xx}/\rho_{xy})^2 + \dots \tag{3}$$

The existence of a linear relation was proposed in a number of papers[3,4]; Laughlin[5] noted that the corrections $\delta\rho_{xy}$ are attributed only to energy dissipation in a 2M layer and, for this reason, are quadratic in ρ_{xx}; the existence of a zero-order term in expansion (3) was admitted in Refs. 6 and 7: $\alpha_0 \sim (a_H/\mathscr{L})$, where $\mathscr{L}$ is the characteristic size of the specimen, and $a_H = (c\hbar/eH)^{1/2}$ is the magnetic length. The last assumption appears to us to be the least plausible, since $a_H/\mathscr{L} \sim (1\text{–}5)\times 10^{-5}$, while the estimate $(\delta\rho_{xy}/\rho_{xy}) \lesssim 10^{-7}$ follows from the experiment.[8]

Experiment. We investigated the range of carrier densities corresponding to the wings of the plateau in ρ_{xy}, where $\nu(\mathscr{E}_F)$ differs from zero and the state of the specimen may be assumed to be uniform[9] ($\nu(\mathscr{E}_F)$ is the energy density of states at the Fermi level). In the experiment we used Si MIS structures with an end channel and orientation of the (100) face; the maximum mobility in the absence of a magnetic field was $\sim 1.6\times 10^4$ cm^2/V·s at $T = 4.2$ K. The voltage dependences of ρ_{xx} and $|\delta\rho_{xy}|$ at the gate voltage V_g, measured at $T = 0.4$ K in a field of $H = 80$ kOe, are illustrated in Fig. 1a.

It is evident in Fig. 1 that in the interval of deviations from the center of the plateau $3\% < N|V_3 - {}^0V_3|/{}^0V_3 \leqslant 12\%$, i.e., a linear relation exists for $|\delta\rho_{xy}/\rho_{xy}| \simeq 10^{-3}\text{–}10^{-5}$ (Fig. 1):

$$\delta\rho_{xy}/\rho_{xy} = \alpha_1(\rho_{xx}/\rho_{xy})\,\mathrm{sign}\,({}^0V_3 - V_3). \tag{4}$$

To compare the experimental data, measured with different currents, the width of the plateau in ρ_{xy} and minima in ρ_{xx} along the V_g axis must be normalized. We used the empirical equation describing the dependence of the width of the plateau on the current and temperature from measurements[9] at $H = 80$ kOe

$$\Delta V_{\text{plateau}} = A(J^c - J_x)(T^c - T) + B\log\delta + C, \tag{5}$$

where $A = 10$ mV/(μA·K), $J^c \simeq 15.3$ μA, $T^c \simeq 2.5$ K, $B \simeq 115$ mV, $C \simeq 480$ mV, and $\delta = |\delta\rho_{xy}/\rho_{xy}|$. According to (5), $\Delta V_{\text{plateau}}$ (0.9 μA) $- \Delta V_{\text{plateau}}$ (9μA) $= 20$ mV; the experimental points ρ_{xx} and $\delta\rho_{xy}$, measured at a current of 0.9 μA and temperature 2.36 K, are shifted toward the center by one-half this magnitude in Fig. 1b. It thus follows from Figs. 1a and 1b that the coefficient $\alpha_1 \simeq 0.3\text{–}0.4$ in empirical dependence (4) does not depend on temperature (in the region $T = 0.4\text{–}2.4$ K) or the current (in the region $J_x = 0.9\text{–}9$ μA).

The dashed line in Fig. 1 shows, for example, the quadratic dependence $\propto (\rho_{xx})^2$.

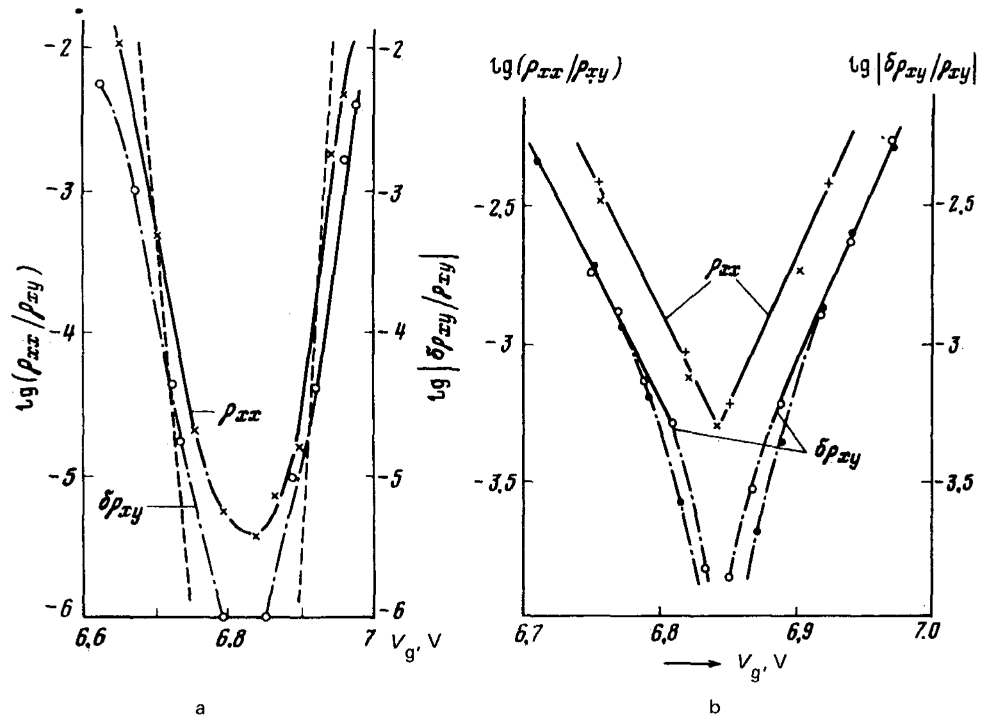

FIG. 1. Modulus of the deviation of the Hall resistance $|\delta\rho_{xy}| = |\rho_{xy} - h/Ne^2|$ and the resistivity ρ_{xx} in the region of the wings of the plateau ($N = 4$, $H = 80$ kOe): a) at $T = 0.45$ K, $J_x = 9\ \mu$A; b) at $T = 2.36$ K, $J_x = 9\ \mu$A (● and ×); with $J_x = 0.9\ \mu$A (O and +) with the width normalized according to Eq. (5).

We also note a geometrical correction $\beta_1(\rho_{xx}/\rho_{xy}) = 10^{-2}(\rho_{xx}/\rho_{xy})$, arising because the Hall angle differs from $\pi/2$ as a result of end effects,[10] which is, first of all, 1.5 orders of magnitude smaller than the measured value α_1 in Eq. (4) and, secondly, it is symmetrical relative to the center of the plateau; therefore, it does not affect result (4). For large deviations $|\delta\rho_{xy}/\rho_{xy}| > 10^{-2}$, linear relation (4) disappears. This should be expected, since the ratio ρ_{xx}/ρ_{xy} is no longer a small parameter in (1) and $\rho_{xx}(V_3)$ approaches the value ρ_{xx}^{max}.

We shall analyze *the reason for linear relation (4)* with the help of relations for two-dimensional tensors. Let us assume that a small deviation $\delta\sigma_{xy} = \sigma_{xy} - Ne^2/h \ll \sigma_{xy}$, as well as nonzero values $\sigma_{xx} \ll \sigma_{xy}$ and $\rho_{xx} \ll \rho_{xy}$, have appeared (σ_{lm} is the conductivity tensor). Then

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx \sigma_{xy}^{-1}\left[1 - \frac{\delta\sigma_{xy}}{\sigma_{xy}} - \left(\frac{\rho_{xx}}{\rho_{xy}}\right)^2\right], \tag{6}$$

It is evident from here that the appearance of finite dissipation ($\rho_{xx} \neq 0$) with $\delta\sigma_{xy} = 0$ would lead to a quadratic relation between $\delta\rho_{xy}$ and ρ_{xx}. Therefore, the reason for linear relation (4), observed experimentally, is the change in the Hall component σ_{xy}.

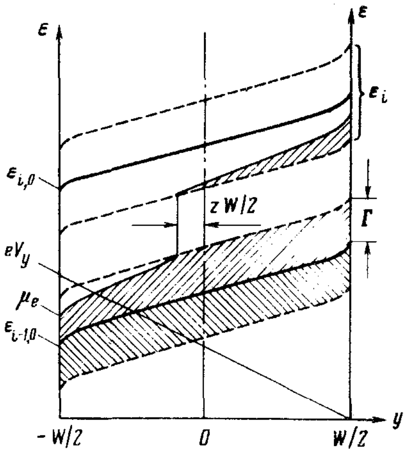

FIG. 2. Schematic distribution of energy of Landau levels $\mathscr{E}_i$, electrochemical potential μ_e, and the Hall potential eV_y over the width of the channel with small deviations from the center of the plateau in ρ_{xy}. The shaded regions of electronic and hole states in the Landau subbands are filled with carriers.

In other words, the basic mechanism for the increase in $|\delta\rho_{xy}|$ and ρ_{xx} in the region of the wings of the plateau is the change in the density of mobile carriers that transport the current $\delta n_{\text{mobile}}/n_H = -(\delta\rho_{xy}/\rho_{xy}) \propto \sigma_{xx} \propto \rho_{xx}$ (here $n_H = 1/2\pi a_H^2$ is the density of states at the two-dimensional Landau level), rather than the dissipation of energy in the two-dimensional layer. This result permits describing the behavior of ρ_{xx} and ρ_{xy} theoretically by finding the number of mobile and localized carriers.

We shall make a *qualitative analysis of the experimental data* by using the idea of localization of carriers in long-period fluctuations of the potential (on a scale a_H). The region of the plateau in this model corresponds[6,11] to dissipation-free transport of current along states lying at the percolation level below the Fermi energy and, as a result, almost completely filled. It is well known[12,11] that the percolation level for a two-dimensional Landau subband $\mathscr{E}_i$ coincides with its average energy $\mathscr{E}_{i,0}$. It is reasonable to assume that the Hall field $\mathscr{E}_{i,0}$ and the current density j_x are constant across the width of the channel.[7] In this case, the change in energy $\mathscr{E}_{i,0}$ over the width of the channel W is eV_y (where V_y is the Hall potential), while the change in the electrochemical potential (Fig. 2) is

$$\Delta\mu_e \equiv \mu_e(y = W/2) - \mu_e(y = -W/2) = eV_y \int_{-W/2}^{W/2} \left(\frac{d\mu_e}{dV_3}\right) E_y\, dy. \tag{7}$$

The number of mobile carriers at the percolation level $\mathscr{E}_{i-1,0}$ and $\mathscr{E}_{i,0}$ is

$$\frac{n_{\text{mobile}}}{n_H} = \frac{1}{W} \int_{-W/2}^{W/2} [f(\mathscr{E}_{i-1,0}, \mu_e) + f(\mathscr{E}_{i,0}, \mu_e)]\, dy. \tag{8}$$

Here f is the Fermi distribution function. Integrating (8) near the center of the plateau

$z \equiv (V_3 - {}^0V_3)/V_y \to 0$ gives

$$\frac{\delta n_{\text{mobile}}}{n_H} \equiv \frac{n_{\text{mobile}} - n_H}{n_H} \simeq z e^{-\frac{\Gamma}{kT}} e^{-\alpha}, \tag{9}$$

while for large deviations from the center of the plateau $|z| \gg 1$, we find

$$\left|\frac{\delta n_{\text{mobile}}}{n_H}\right| \simeq \frac{1}{2\alpha} e^{-\frac{\Gamma}{kT}} e^{(-\alpha + |\alpha z|)}. \tag{10}$$

In Eqs. (9) and (10) Γ is the energy width of the Landau subband, $\alpha = (dn/dV_3)V_y/[2\nu(\mu_e)kT]$ ($\alpha < 0$ for the case in Fig. 2).

Equation (9) describes qualitatively correctly the behavior of $\delta\rho_{xy}$ for $|V_3 - {}^0V_3| \to 0$ (Fig. 1), but the large error in the experimental data in this region makes it difficult to compare them with Eq. (9). In the region of large deviations from the center of the plateau $|V_3 - {}^0V_3| \gtrsim 0.15$ V (or $|\delta\rho_{xy}/\rho_{xy}| > 10^{-3}$), the index α of the exponential dependence of $\delta\rho_{xy}(V_3)$ increases with decreasing temperature (See Fig. 1), consistent with Eq. (10). The proportionality between the exponent in(10) and the current $\alpha \propto V_y \propto j_x$ also agrees with empirical equation (5). However, over a rather wide intermediate region $V_y < |V_3 - {}^0V_3| < 0.15$ V (or $|\delta\rho_{xy}/\rho_{xy}| = 10^{-3}$–$10^{-5}$), according to Eq. (10), $|\delta\rho_{xy}| \infty \exp[A(V_3 - {}^0V_3)/T]$, while from experiment (Fig. 1b) it follows that $\delta\rho_{xy} \propto \exp[B(V_3 - {}^0V_3)T]$. An analogous deviation from the activation temperature dependence was found in Ref. 9 and the reason for it is not understood.

Calculating the number of localized carriers for $z > 1$ in an analogous manner, we obtain an equation for the width of the plateau on the density scale

$$\Delta n_{\text{plateau}}(T, J_x) \simeq \frac{n_H}{\Gamma}[\Gamma + kT \ln\delta - 2kT\alpha]\left[1 - \frac{\pi^2}{3}\left(\frac{kT}{\Gamma}\right)^2\right],$$

which, according to (5), describes the narrowing of the plateau as a function of current and temperature[9] ($\delta = |\delta\rho_{xy}/\rho_{xy}|$ is the criterion for the width of the plateau).

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