Quantized Hall effect due to charge density waves

Yu. A. Bychkov and É. I. Rashba
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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A theory of quantized Hall conductivity σ_H which takes into account the interelectronic interaction via a charge density wave (CDW), is constructed. σ_H increases linearly with increasing degree of filling ν with the exception of rational points $\nu = r/p$ with small p, where a gap arises in the spectrum, while σ_H exhibits a singularity, becoming a multiple of e^2/h.

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The discovery of quantization of Hall conductivity (QHC) σ_H in two-dimensional (2D) conductors generated interesting problems and attracted attention to this field. von Klitzing *et al.*[1] discovered integer QHC $\sigma_H = \sigma_{yx} = \nu e^2/h$, where $\nu = N$ is an integer equal to the number of filled Landau levels (N also includes the spin index). Tsui *et al.*[2] discovered fractional QHC with $\nu = 1/3, 2/3$, while Stromer *et al.*[3] discovered QHC with $\nu = r/p$, $p = 3,5,7$. Fractional ν correspond to partial filling of the upper Landau level. In all cases, QHC is observed in the form of a plateau on the resistance curves ρ_{xy}, equal in height to $h/e^2\nu$, and deep dips on the curves ρ_{xx}. In addition, $\rho_{xx} \ll \rho_{xy}$ and $\sigma_{yx} \approx (\rho_{xy})^{-1}$.

The main features of the mechanism of integer QHC has already been interpreted. The mechanism responsible for fractional QHC remains puzzling. The only position that is generally accepted is that this effect is due to interelectronic ($e - e$) interaction: This is the only interaction that can give rise to a gap in the spectrum, which, as assumed, will be manifested in $\sigma_H(\nu)$ as a plateau. In all three papers known to us on fractional QHC (Yoshioka *et al.*,[4] Laughlin,[1),5] and Tao and Thouless[8]), σ_H is not calculated, while the energy is determined by different methods, falling outside the framework of the Hartree-Fock (HF) method. In view of the complexity of the problem, assumptions are made in these papers whose consequences are difficult to estimate. We shall calculate the electronic energy by the HF method via the self-consistent CDW field and we shall find $\sigma_H(\nu)$.

The appearance of CDW in 2D systems at temperatures $T_{B\,\mathrm{CDW}\pi} \sim [e^2\nu(1-\nu)/\epsilon\lambda(B)]$, where ϵ is the dielectric constant, $\lambda(B) = (c\hbar/eB)^{1/2}$ is the magnetic length, and B is the magnetic length, was predicted by Fukuyama *et al.*[9] We assume that CDW form a two-dimensional lattice. To simplify the equations, we assume a rectangular unit cell with edges a and b. The CDW lattice and the magnetic lattice (with cell area $2\pi\lambda^2$) must be commensurable:

$$ab = \frac{p}{q}\, 2\pi\lambda^2(B), \tag{1}$$

p and q are prime numbers with respect to each other. We shall assume that the field B is strong: $\hbar\omega_c \ll e^2/\epsilon\lambda$, where $\omega_c = eB/m^*c$ is the cyclotron frequency. When the CDW

potential, which is of the order of $e^2/\epsilon\lambda$, can be viewed in comparison with $\hbar\omega_c$ as a perturbation. For this reason, the wave functions $\psi_{\mathbf{k}nN}(\mathbf{r})$, which conform to the symmetry of the magnetic lattice, can be constructed as linear combinations of the functions $\psi_{p_yN}(\mathbf{r})$, chosen in the Landau gauge, with one value of N:

$$\psi_{\mathbf{k}n}(\mathbf{r}) = \sum_{l=-\infty}^{\infty} \exp\left\{ i \frac{2\pi}{b} (n+lp)(y+\lambda^2 k_x) \right\} \psi_{k_y}\left(x - \lambda^2 k_y - \frac{2\pi\lambda^2}{b}(n+lp)\right) \quad (2)$$

(the index N is dropped). The new quantum numbers $\mathbf{k}$ and n are defined by the relations

$$\begin{aligned} &p_y = k_y + (n+lp)Q_{y0}, \quad 0 \leqslant n < p, \\ &0 \leqslant k_x \leqslant Q_{x0}/q = 2\pi/qa, \quad 0 \leqslant k_y \leqslant Q_{y0} = 2\pi/b. \end{aligned} \quad (3)$$

Thus there exist p functions $\psi_{\mathbf{k}n}(\mathbf{r})$, defined in the new Brillouin zone (3); $\psi_{\mathbf{k},n+p} = \psi_{\mathbf{k}n}$. The eigenfunctions $\psi_{\mathbf{k}\alpha}(\mathbf{r})$ have the form

$$\psi_{\mathbf{k}\alpha}(\mathbf{r}) = \sum_n C_{\alpha n}(\mathbf{k})\, \psi_{\mathbf{k}n}(\mathbf{r}). \quad (4)$$

For them, the condition of periodicity in $\mathbf{k}$ has the form $\psi_{\mathbf{k}+Q,\alpha}(\mathbf{r}) = \psi_{\mathbf{k}\alpha}(\mathbf{r}) \exp\{i\theta_\alpha(\mathbf{k},\mathbf{Q})\}$.

In the HF approximation, all $\vartheta_{\alpha n}$ are related by the following system of equations, which determine the p branches of the spectrum (minibands), into which each Landau level is split:

$$\begin{aligned} E_{\mathbf{k}} C_{\alpha n}(\mathbf{k}) = \sum_{Q_x} \sum_{n'=0}^{p-1} \sum_{l=-\infty}^{\infty} U(Q) g(\mathbf{Q}) \exp\left\{ -ik_x \frac{2\pi\lambda^2}{b}(n'+lp) \right\} \\ \times \exp\left\{ iQ_x k_y \lambda^2 + iQ_x \frac{2\pi\lambda^2}{b}\left(n - \frac{n'}{2} - \frac{l}{2}p\right) \right\} C_{\alpha, n-n'}(\mathbf{k}). \end{aligned} \quad (5)$$

Here $E_{\mathbf{k}}$ is the energy,

$$U(Q) = \frac{1}{2\pi} V(Q)\, w_N^2(Q^2) - \int_{-\infty}^{\infty} \frac{d^2k}{(2\pi)^2} e^{i\lambda^2 \mathbf{k}\mathbf{Q}} V(k) w_N^2(k^2) \quad (6)$$

is the HF potential, $V(k)$ is the Fourier component of the potential $V(r)$ of the effective $e-e$ interaction (which is different for 2D systems), while $w_N(k^2) = L_N(z)e^{-z/2}$, $z = \lambda^2 k^2/2$, and L_N are Laguerre polynomials. The function $g(\mathbf{Q})$, which is related to the electron density distribution, is determined from the self-consistency condition. This condition holds if the expression for the electron density distribution is written in terms of the operator $\hat{\Psi}(\mathbf{r}) = \sum_{\mathbf{k}\alpha} \psi_{\mathbf{k}\alpha}(\mathbf{r})\hat{A}_{\mathbf{k}\alpha}$

$$< \hat{\Psi}^+(\mathbf{r})\,\hat{\Psi}(\mathbf{r}) > = \frac{1}{2\pi} \sum_{\mathbf{Q}} w_N(Q^2)\, g(\mathbf{Q}) \exp(i\mathbf{Q}\mathbf{r})$$

$$= \sum_{\alpha, n} \sum_{\mathbf{Q}} \int \frac{d^2k}{(2\pi)^2} \exp\left\{ - i\lambda^2 \left(\frac{Q_x Q_y}{2} + k_y Q_x \right) - i\theta_\alpha(\mathbf{k}, \mathbf{Q}) + i\mathbf{Q}\mathbf{r} \right\}$$

$$\times f(E_{\mathbf{k}\alpha})\, w_N(Q^2) C_{\alpha n}(\mathbf{k})\, C^*_{\alpha n}(\mathbf{k}+\mathbf{Q}), \tag{7}$$

$f(E)$ is the Fermi function. Equation (7) closes the system and permits finding the total energy $E = E(\nu, p, q)$. Its minimum with respect to p and q[2)] with $\nu = \text{const}$ determines $p = p(\nu)$ and $E = E(\nu)$ (it is logical to assume that $q_{\min} = 1$, and we make this assumption). It is significant that ν was treated above as an independent parameter, which could affect the dimensions of CDW only via $p_{\min}$. In addition, each cell has an arbitrary (irrational) number of electrons. This is natural, since the CDW lattice is a quantum lattice (in contrast to the classical Wigner crystal). If $\nu \to r/p$ (r is an integer), then the cell decreases in size in k space. Simultaneously, new minigaps open up in it and the number of minibands increases to p. For $\nu = r/p$, r of them are completely filled. In addition, the energy $E(\nu)$ must have a singularity: a local minimum. It is for these values of ν that singularities (a plateau) were observed in Refs. 2 and 3.

It seems to us that the HF method can explain a number of features of fractional QHC. The opinion that it is completely inapplicable was formed in the literature based on calculations[10,11] which did not reveal any singularities at $\nu = 1/3$ on the curve $E(\nu)$. However, in Refs. 10 and 11, $E(\nu)$ was calculated with a variable CDW cell size, connected too rigidly with ν.

We shall calculate the conductivity σ_H in the single-particle approximation, i.e., we shall assume that the CDW lattice is stationary as a result of pinning and we shall exclude the contribution of the corresponding collective mode. We shall assume, however, that the lattice is mobile "in the small," i.e., it is capable of rearranging itself as ν varies. In the calculations, we shall make use of the elegant work of Thouless *et al.*[12] It follows from Kubo's equation that

$$\sigma_H = -\frac{e^2}{\hbar} \lambda^2 \sum_{\alpha} \int \frac{d^2k}{(2\pi)^2} f(E_{\mathbf{k}\alpha}) (\mathrm{rot}_{\mathbf{k}} \mathbf{A}_\alpha(\mathbf{k}))_z, \tag{8}$$

where

$$(\mathbf{A}_\alpha(\mathbf{k}))_j = \frac{i}{2} \int d^2r \left(u^*_{\mathbf{k}\alpha} \frac{\partial u_{\mathbf{k}\alpha}}{\partial k_j} - u_{\mathbf{k}\alpha} \frac{\partial u^*_{\mathbf{k}\alpha}}{\partial k_j} \right). \tag{9}$$

The integration in (8) and (9) is bounded by the cell of the direct and reciprocal CDW lattices, while $u_{\mathbf{k}\alpha} = \psi_{\mathbf{k}\alpha} \exp(-i\mathbf{k}\mathbf{r})$. Using (2) and (4), we obtain

$$(\mathbf{A}_\alpha(\mathbf{k}))_j = k_y \delta_{jx} + \frac{i}{2} \sum_n \left\{ C^*_{\alpha n} \frac{\partial C_{\alpha n}}{\partial k_j} - C_{\alpha n} \frac{\partial C^*_{\alpha n}}{\partial k_j} \right\}. \tag{10}$$

Assume, at first, that ν does not approach any number r/p with large p, while $\nu(1-\nu)$ is not too small. Then both cells, (the magnetic and CDW) coincide, $p = q = 1$, and

$C_{11} = 1$ is the only remaining coefficient. Then, only the first term remains in (10), and

$$\sigma_H = (e^2/h)\nu. \tag{11}$$

This result is nontrivial, since electrons move in the field of the CDW. If $\nu(1-\nu)$ is small, so that $T_{B\,\mathrm{CDW}H}(\nu) < T$, then (11) is also valid.

If $\nu = r/p$ with small p, then the CDW lattice is rearranged and its Brillouin zone decreases in size by a factor of p. In this case, all coefficients $C_{\alpha n}$ are of comparable magnitude. It is shown in Ref. 12 that at such points

$$\sigma_H = (e^2/h)s, \tag{12}$$

where $s(\rho,\alpha)$ is a positive or negative integer, which depends on the specific model. But, if $\nu = m$ (m is an integer), then $\sigma_H = (e^2/h)m$, which also agrees with (11).

It is clear from the preceding disucussion that the CDW can be restructed with a change in p, which occurs with $\nu \neq r/p$.

The basic results are as follows. Far away from the points r/p (with small p), as well as for small $\nu(1-\nu)$, the conductivity σ_H follows the "classical" law (11). For $\nu = m$, $\sigma_H(\nu)$ is also continuous (the plateau is formed due to an external mechanism, a "reservoir"[13]). For $\nu = r/p$, the HF approximation leads to opening of gaps in the spectrum and to singularities in $\sigma_H(\nu)$, large peaks up to the integer-valued $\sigma_H(e^2/h)s$, positive or negative. It is unclear whether these peaks are attributable to defects in the HF method, which gives a single-particle description of the transport phenomena, or whether they are real effects, which have not yet been observed due to imperfection of the specimens. The key role here could be played by the partial pinning of the CDW lattice, retaining local mobility sufficient for local restructuring of CDW and formation of gaps, but breaking down the long-range correlation [to which Eq. (12) could be more sensitive] and giving rise to dissipation. If remains unclear how the external manifestations of the characteristics described above change under conditions of the mosaic structure of CDW resulting from pinning.

[1)]The classification of levels of finite-electron clusters, developed in Refs. 6 and 7, is used in Ref. 5
[2)]It can also be generally determined from the shape of the cell.

[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[2]D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[3]H. L. Stormer, A. Chang, D. C. Tsui, J. C. M. Hwang, A. C. Gossard, and W. Wiegman, Phys. Rev. Lett. **50**, 1953 (1983).
[4]D. Yoshioka, B. I. Halperin, and P. A. Lee, Phys. Rev. Lett. **50**, 1219 (1983).
[5]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[6]Yu. A. Bychkov, S. V. Iordanskiĭ, and G. M. Eliashberg, Pis'ma Zh. Eksp. Teor, Fiz. **33**, 152 (1981) [JETP Lett. **33**, 143 (1981)]; Poverkhnost', No. 10, 33 (1982).
[7]R. B. Laughlin, Phys. Rev. B **27**, 3383 (1983).
[8]R. Tao and D. J. Thouless, Preprint, University of Washington (1983).
[9]H. Fukuyama, P. M. Platzman, and P. W. Anderson, Phys. Rev. B **19**, 5211 (1979).
[10]D. Yoshioka and H. Fukuyama, J. Phys. Soc. Jpn. **47**, 394 (1979).
[11]D. Yoshioka and P. Lee, Phys. Rev. B **27**, 4986 (1983).
[12]D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).
[13]G. A. Barraf and D. C. Tsui, Phys. Rev. B **24**, 2274 (1981).

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