Triangle anomaly at a nonzero temperature in quantum chromodynamics

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(Submitted 22 June 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **38**, No. 4, 215–217 (25 August 1983)

An analysis based on the Adler-Bell-Jackiw equation for a nonzero temperature shows that the temperature (T_F) at which the chiral symmetry is restored is greater than or equal to the deconfinement temperature T_c. At temperatures in the interval $T_c < T < T_F$, massless quarks acquire a nonzero dynamic mass.

PACS numbers: 12.35.Cn, 11.30.Rd

Using the methods of quantum field theory[1–5] to study relativistic matter at a nonzero temperature is important for reaching an understanding of the early universe and for other astrophysical applications.[5] There is particular interest in phase transitions in grand unified models and in quantum chromodynamics. Polyakov[6] and Susskind[7] have shown that a phase transition from a confinement phase to a phase of free quarks occurs at a certain temperature T_c in quantum chromodynamics. On the other hand, in a quantum chromodynamics with n massless quarks, the chiral symmetry group $G = SU_{L-R}(h)$ is spontaneously violated at $T = 0$ (Ref. 8). The chiral symmetry is restored at a certain temperature T_F in quantum chromodynamics.[9] We are thus extremely interested in the relationship between the deconfinement temperature T_c and the temperature at which the chiral symmetry is restored, T_F. Working from the Adler-Bell-Jackiw equation[11] and the assumption of confinement in quantum chromodynamics, 't Hooft recently showed[10] that a violation of chiral symmetry is unavoidable in quantum chromodynamics (with $n \geqslant 3$) at a zero temperature.

In the present letter we show that the Adler-Bell-Jackiw equation, originally derived for $T = 0$, remains the same in form at a nonzero temperature. As a result, 't

 0021-3640/83/160255-03$01.00

Hooft's analysis is also valid at nonzero temperatures. We will show that

$$T_F \geqslant T_c . \tag{1}$$

Furthermore, it follows from our analysis that at temperatures $T_c < T < T_F$ massless quarks acquire a nonzero dynamic mass.

We recall that at $T = 0$ the Adler-Bell-Jackiw equation in a quantum chromodynamics with n massless quarks is

$$k_\mu T^{\mu\nu\alpha}_{abc}(p_1, p_2) = N_c \, 4\pi^2 \epsilon^{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta \, Tr\,[\{T_a T_b\}_+ T_c\,] ,$$

$$T^{\mu\nu\alpha}_{abc}(p_1, p_2) = (2\pi)^4 \int d^4x\, d^4y\, e^{-ip_1 x - ip_2 y}$$

$$< 0|\, T(J^\mu_a(x), J^\nu_b(y)\, J^\alpha_{5c}\ (0))\,|\,0> , \tag{2}$$

$$J^\mu_a = \bar\psi j^\mu T_a \psi\ ,\ \ J^\mu_{5c} = \bar\psi j^\mu j^5 T_c \psi\ ,$$

where $N_c = 3$ is the number of colors, and T_a are the generators of the SU(n) group. The tensor $T^{\alpha\mu\nu}_{abc}$ satisfies the Ward identities

$$p_{1\mu} T^{\mu\nu\alpha}_{abc} = p_{2\nu} T^{\mu\nu\alpha}_{abc} = 0 \tag{3}$$

and the condition for Bose symmetry, $T^{\alpha\mu\nu}_{abc}(p_1, p_2) = T^{\nu\mu\alpha}_{abc}(p_2, p_1)$. Analyzing Eq. (2), and adopting the hypothesis of quark confinement, 't Hooft showed that at $n > 2$ the chiral $SU_{L-R}(n)$ symmetry should be spontaneously violated. 't Hooft's proof was based on the argument that an anomaly could arise on the right side of Eq. (2) only if the spectrum of the theory contained massless particles. If the chiral symmetry in quantum chromodynamics were exact, certain baryons would have a zero mass. At $n > 2$, however, incorporating massless baryons does not reproduce[10] anomalous equation (2). We note that in proving the spontaneous violation of chiral symmetry 't Hooft used a decoupling condition in addition to Eq. (2), and this coupling condition was recently criticized by Preskill and Weinberg.[12] With $n = 3K$ (where K is an integer), however, 't Hooft's proof follows directly from Eq. (2) without the help of the decoupling condition.

If we consider a quantum field theory with a temperature, we lose relativistic invariance. To formally restore relativistic invariance it is convenient to introduce an external temperature 4-vector n^μ_T ($n^2_{\mu T} = T^2$). In the case $T \neq 0$ all the Green's functions depend on the external 4-temperature n^μ_T. It is not difficult to see that the equations of motion are independent of the temperature. Furthermore, a direct calculation shows that Eq. (2) and the Ward identities (3) remain the same at $T \neq 0$. The fact that Eq. (2) also holds at a nonzero temperature is a direct consequence of the circumstance that Eq. (2) stems from the linear divergence of the corresponding single-loop integrals. Since the introduction of a nonzero temperature does not introduce any ultraviolet divergences not already present at $T = 0$, Eq. (2) remains the same, except that on its right side we have

$$T^{\mu\nu\alpha}_{abc}(p_1, p_2, n_T) = \int\limits^{n_{\mu T} = (T, 0\,0\,0)} \langle 0 | T(J^{\mu}_{a}(x) J^{\nu}_{b}(y) J^{\alpha}_{sc}(0))$$

$$|0\rangle_T e^{-ip_1 x - ip_2 y},$$

where

$$\langle 0 \rangle_T = \mathrm{Sp}(e^{-H/T} O) / \mathrm{Sp}(e^{-H/T}),$$

H is the Hamiltonian of the system, and the Sp (trace) is taken over all states. 't Hooft's analysis can thus be extended intact to the case $T \neq 0$. It follows that we immediately know that the temperature at which the chiral symmetry is restored, T_F, cannot be lowered than the deconfinement temperature T_c [inequality (1)]. We might note that Kogut *et al.*[13] found $T_F > T_c$ through a numerical integration using the lattice approximation. At $T_c < T < T_F$, the massless pseudoscalar mesons (Goldstone particles) of the $SU_{L-R}(n)$ group will contribute to the right side of Eq. (2). If we are to avoid an additional contribution to the right side of Eq. (2), the massless quarks must acquire a nonzero dynamic mass at $T_c < T < T_F$.

I thank V. A. Matveev and A. N. Tavhelidze for useful discussions.

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Translated by Dave Parsons
Edited by S. J. Amoretty