# Hadron bubbles in nuclear matter

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Nonlinear effects in the interaction of hadrons with a nucleus are analyzed. It is shown that $K^+$ mesons form bubbles in nuclear matter which are similar to electron bubbles in liquid helium. Charged pions produced in collisions of heavy relativistic ions may collect and form droplets $\sim$5–7 Fm in size containing $\sim 10^2$ particles.

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Extensive information has now been accumulated on the low-energy interactions of hadrons with nuclei. The experimental data available can be reproduced satisfactorily in a linear approach, in which the hadron-nucleus interaction $H_{\rm int}$ is described by an optical potential $U_a(\mathbf{r})$, i.e., $H_{\rm int}(\mathbf{r}) = U_a(\mathbf{r})\varphi_a^2(\mathbf{r})$, where $\varphi_a(\mathbf{r})$ is the hadron wave function. This potential is usually introduced as a function of the unperturbed nuclear density (the equilibrium value is $\rho_0 = 0.17\ \mathrm{Fm}^{-3}$), and the parameters are determined by fitting the experimental data available. With the optical potential $U_a(\mathbf{r})$ we can also associate a hadron field $V(\mathbf{r})$ which acts on nucleons: $H_{\rm int} = U_a(\mathbf{r})\varphi_a^2(\mathbf{r}) = \rho_0(\mathbf{r})V(\mathbf{r})$, i.e., $V(r) \sim U_a(r)\varphi_a^2(r)/\rho_0 \sim (U_a r_0^3/a^3)Q$, where $r_0$ is the average distance between the particles in the nucleus, $a$ is the characteristic dimension of the hadron wave packet, and $Q$ is the number of hadrons in the packet. If $V \sim \mathscr{E}_F$ ($\mathscr{E}_F$ is the Fermi energy of the nucleons), the nucleon density $\rho(\mathbf{r})$ changes significantly; then there is also a change in the nuclear field $U_a(\mathbf{r})$ acting on the hadrons, so that the linear approach cannot be taken. Several nonlinear effects in nuclear matter, associated primarily with solitons, were studied in Refs. 1–3. In the present letter we analyze yet another effect: the localization of hadrons in nuclear matter which results from the rather strong interaction of the hadrons with nucleons. A similar effect is observed for electrons and positive ions in liquid helium. When an electron, whose interaction with a neutral He atom is very strong and repulsive ($U_e > 0$), enters liquid He it forms a "bubble": a cavity almost devoid of helium. In contrast, a positive He ion, which attracts the neutral He atom ($U_{\rm He} < 0$), becomes covered with a crust of solidified helium.[4,5]

Why do hadron bubbles form during a repulsive hadron-nucleus interaction, and how do we calculate their characteristics? As long as a hadron is moving in the cavity which it has produced, where there is no matter, its (short-range) interaction with the particles of the nuclear medium does not occur. As soon as the hadron finds itself in the medium, however, this interaction commenses, and it causes a sharp increase in the hadron energy, $\delta E \sim \langle H_{\rm int}\rangle \simeq \int U_a \varphi_a^2 d^3r = U_a$. Accordingly, the cavity is a potential well for the hadron. When the hadron occupies one of the discrete levels with energy $\varepsilon_\lambda$ (a necessary condition here is $U_a > U_c = \pi^2/8m_a a^2$), its motion becomes localized, and its energy decreases. If this energy decrease exceeds the energy expended on the formation of the cavity in the nuclear medium $\delta E_S \simeq 4\pi\sigma a^2$, where $\sigma$ is the surface-tension coefficient of the nucleus), then the state in question is stable in the nuclear matter, and in a real nucleus its width will be determined by a tunneling effect: the tunneling of a hadron through the surface barrier. The condition for a minimum of the total energy $E = 4\pi\sigma a^2 + \Sigma\varepsilon_\lambda + E_{\rm int}(Q)$ determines the optimum size $a$ of the bubble, which contains $Q$ hadrons ($E_{\rm int}$ is their interaction energy). Among the hadrons for which the potential $U_a$ is known, only the $K^+$ meson has a sufficiently strong repulsive interaction with nucleons. Experiments by Barns[6] have revealed $U_{K^+} \simeq 55$–$60$ MeV for kaons with $P_K \sim 800$ MeV (for slower kaons, it increases, to 70–80 MeV at $P_K \sim 0$). Simple variational calculations show that stable bubbles exist if $U_a > 11.5\sqrt{\sigma/mQ}$; hence, the $K^+$ bubbles become stable at $Q = 2$. If a two-meson bubble does in fact exist, the implication would be that a strong effective attraction arises between two kaons moving relatively slowly in a nuclear medium, although their interaction in a vacuum may in fact be repulsive. Because of the quantization of the translational motion of the center of mass of the bubble in a finite system, a certain band of resonant compound-nucleus levels should be associated with the bubble.

How can we observe bubble resonances? Let us consider the inelastic scattering of kaons by nuclei, $K^+ + (A,Z) \to (A-1, Z-1) + \lambda^0 + K^+ + K^+$, accompanied by the production of a $\lambda^0$, $K^+$ pair. Two-meson bubbles form if the $K$ mesons have a low relative momentum. Accordingly, for small angles between the emitted kaons with energies $E_K \leqslant U_K$ the cross section for this reaction should have the typical resonance form, similar to that which usually arises during the production of particles that interact strongly in the final state.

A nucleus is a two-component system. An extremely interesting case is that in which the hadrons, repelled from, say, protons, are attracted to neutrons. If a localized hadron state exists, then only neutrons will remain in the hadron droplet. Let us examine this situation in more detail for the case of $\pi$ mesons. The Hamiltonian of a pion in a nucleus is usually written in the form $H_\pi = k^2/2m_\pi + U_\pi^S + U_\pi^P$, where

$$U_\pi^s(r) = -\frac{2\pi}{m_\pi}\left[\left(1+\frac{m_\pi}{M_N}\right)(b_0\rho_0 x + b_1\rho_0 y T_3) + \left(1+\frac{m_\pi}{2M_N}\right)(B_0\rho_0^2 x^2 + B_1\rho_0^2 xy T_3)\right]$$

$$U_\pi^P(r) = -\frac{\hat{k}^2}{2m_\pi}\frac{\alpha}{1+{}^1/_3\,\alpha} + \gamma(\rho)\frac{\hat{k}^4}{m_\pi^3}, \qquad (1)$$

$x = (\rho_n + \rho_p)/\rho_0$, $y = (\rho_n - \rho_p)/\rho_0$, $T_3 = 1$ for the $\pi^-$ meson, and

$$\alpha = 4\pi\left[\left(1+\frac{m_\pi}{M_N}\right)^{-1}(c_0\rho_0 x + c_1\rho_0 y) + \left(1+\frac{m_\pi}{2M_N}\right)^{-1}(C_0\rho_0^2 x^2 + C_1\rho_0^2 xy T_3)\right].$$

The sign of $\gamma(\rho)$ is positive, for otherwise the pion-nucleon system will be unstable; its magnitude is[7] $\gamma \lesssim 0.1$. The parameters of $U_\pi$ vary from paper to paper. In Ref. 8, for example, we find $b_0 = -0.046$ Fm, $b_1 = -0.13$ Fm, $c_0 = 0.66$ Fm$^3$, $c_1 = 0.43$ Fm$^3$, $C_0 = 0.29$ Fm$^6$, $C_1 = 0$, and $B_0 \approx B_1 = 0$, while in Ref. 9 we find $b_0 = -0.023$ Fm, $b_1 = -0.11$ Fm, $c_0 = 0.54$ Fm$^3$, $c_1 = 0.44$ Fm$^3$, and $B_0 \simeq -0.3$ Fm$^6$. We see from these data that the $s$-wave interaction of pions with nucleons is not very important in symmetric nuclear matter ($N = Z$), while in a neutron (or a proton) medium the well for $\pi^+$ ($\pi^-$) mesons increases by some 30–40 MeV. Accordingly, if a sufficient number of $\pi^+$ ($\pi^-$) mesons is produced in a small volume of nuclear matter, then by attracting neutrons (or protons) they may form a hadron droplet. Its radius and also the internal concentration of neutrons and protons are determined by the minimum of the total energy

$$E(Q) = \frac{\beta y^2}{2} \frac{a^3}{r_0^3} + \epsilon_\pi Q + \frac{3}{4\pi} \lambda_{\pi\pi} \frac{Q^2}{a^3} . \tag{2}$$

Here $\lambda_{\pi\pi} = 0.05 \pm 0.05 m_\pi^{-2}$ (Ref. 10) is the scattering length for pions with an isospin $T = 2$. Expression (2) takes into account the circumstance, revealed by calculations, that the changes in the density $\rho_+ = \rho_n + \rho_p$ in the droplet are slight, and the energy of a symmetric nucleus can be described by a quadratic parabola with $\beta \simeq 55$ MeV up to $|y| \sim 1$. [Near the point $|y| = 1$ we must use a more accurate equation of state and take the particular form of $U_\pi$ into account, so that (2) is good only for an estimate here.] We can seek a solution of (2) by assuming $U_\pi$ to be a square well. This solution has the important property that it exists only at $Q > Q_c$; if $Q = Q_c$, then only neutrons remain inside a $\pi^+$ droplet, while only protons remain inside a $\pi^-$ droplet. Calculations with the parameters from Ref. 8 yield $Q_c \simeq 200$. The radius of an $s$-droplet of this type is $a_c = 5$ Fm, and the binding energy per meson is about 2.5 MeV.

Pion droplets can arise in nuclear matter as a result of not only the $s$ terms but also the $p$ terms in (1). Ericson and Myhrer[11] showed that a pronounced $p$ attraction in $U_\pi$ could cause a binding of pions, since the sign of $\hat{k}^2$ in $H_\pi$ becomes negative in nuclear matter with a density $\rho \sim \rho_0$.

It is easy to see that the binding energy of a pion in a nucleus increases when the density $\rho$ or the ratio $N/Z$ increases, and in principle the loss in the nuclear part of the energy can be offset by a gain in the pion part. As usual, at some finite value of $a_c$ the energy of the meson state which arises in the discrete spectrum does not decrease with increasing droplet radius, but instead increases, because of the negative sign of the coefficient $(2\mu_{\rm in})^{-1}$ of the $\hat{k}^2$ term in $H_\pi$. Consequently, the radius of the $p$ droplet is fixed, $a = a_c$ (for given values of $x$ and $y$), and the energy is $\varepsilon_\pi = \varepsilon_{\min}$. Solution of the Schrödinger equation with Hamiltonian (1) yields $\varepsilon_{\min} = U_\pi^S m_\pi^3 / 16|\mu|_{\rm in}^2 \gamma$. Analysis shows that (1) the minimization problem has a solution if $\gamma < \gamma_m \simeq 0.1$ and (2) there are no solutions if $Q < Q_c$ (as before), and at $Q = Q_c \sim 10^2$ the droplet is "pure", $|y| = 1$, but now $\pi^-$ mesons combine with neutrons, while $\pi^+$ mesons combine with protons. Since there is no volume absorption of pions in a $p$ droplet, the lifetime of this nuclear "centaur" is determined exclusively by the capture of pions by a nucleus from the surface of the droplet, so that this lifetime is significantly shorter than the nuclear lifetime. The charge $Q$ decreases in the course of such processes, ultimately falling below $Q_c$; the droplet then explodes. We note that if $\gamma \leqslant 0.02$, the value of $\varepsilon_{min}$ is comparable to $m_\pi$, and a phase transition from a homogeneous phase to a two-phase droplet situation can occur in the nuclear matter. This question will be studied separately.

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# Energy limits of Bragg diffraction of charged particles in a single crystal

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It is shown that diffraction of fast charged particles in a single crystal vanishes with increasing particle energy. The temperature dependence of the effect is found.

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**1.** Bragg diffraction of charged particles consists of their reflection from a system of crystallographic planes by small angles. If we are not interested in channeling,[1,2] then the angle of reflection is greater than the Lindhard critical angle $\theta_L$. The angle of reflection due to diffraction is related to the reciprocal lattice vector $g$ by the relation

$$\theta \approx 2\sin(\theta/2) = (\hbar g/p).$$

The condition $\theta/2 > \theta_L = \sqrt{2EV_0}/pc$ means that diffraction occurs only for the values

$$g > g_{min} = \frac{1}{\hbar c}\sqrt{8EV_0}, \qquad (1)$$

where $V_0$ is the effective potential barrier of the channel. Thus the region of Bragg diffraction has a lower limit.

**2.** On the other hand, the existence of incoherent quasielastic scattering by thermally induced fluctuations of the potential limits the region of existence of Bragg scattering from above. The necessity of exact orientation of the velocity of the particles relative to the crystallographic planes for Bragg diffraction leads to the fact that under

  