Lifetime of the 2*p* state and Lamb shift in the hydrogen atom

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The ratio of the frequency (ν) of the transition ($2s_{1/2}, F=0$)–($2p_{1/2}, F=1$) to the constant (γ) of the decay of the $2p_{1/2}$ state of the hydrogen atom was measured in experiments which have been described elsewhere. The constant γ was calculated within ~ 0.3 *ppm*. The corresponding values of ν and of the Lamb shift are $\nu = 909.8934(19)$ MHz and $\delta\,(H, n=2) = 1057.8514(19)$ MHz.

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In measurements of the Lamb shift in the hydrogen atom by an atomic-interferometer method[1,2] it has been found possible to reduce the experimental error to 1.9 kHz. This error is several times smaller than the estimated errors of the most accurate of the other reported values of the Lamb shift, both experimental[3,4] and theoretical.[5,6]

This method has been used to measure the interference of phase-shifted $2p_{1/2}$ states (with a projection $F=1$ of the total hyperfine-structure angular momentum) of a hydrogen atom starts in the $2s_{1/2}$ ($F=0$) metastable state and moves at a constant velocity v through two plane capacitors with a longitudinal electric field separated by a field-free gap of adjustable length l.

As shown in Ref. 2, the interference effect is described by the function

$$\cos\frac{2\pi\nu}{v}\,(1-v^2/c^2)^{1/2}\,l, \tag{1}$$

which contains the frequency (ν) of the transition ($2s_{1/2}, F=0$) – ($2p_{1/2}, F=1$), in which we are interested. The velocity of the particles is determined from the decay curve of the atom in the $2p$ state,

$$\exp\left\{-\,\gamma\frac{l}{v}\,(1-v^2/c^2)^{1/2}\right\}, \tag{2}$$

and expressed in terms of the decay constant $\gamma = 1/\tau$ of this state. The transition frequency ν is then determined by comparing the experimental and theoretical [expression (1)] interference curves. In this manner, the ratio ν/γ was actually measured within an error ~ 2 *ppm* in the experiments of Refs. 1 and 2.

In order to determine ν highly accurately (~ 1 *ppm* $= 10^{-6}$) we must therefore know the decay constant γ with at least the same accuracy. We might note that in extracting the frequency ν from this experiment the error in γ is a systematic error for ν.

Sufficiently accurate experimental data on the decay constant are not available, so a theoretical value was calculated. In the calculation of γ in Ref. 2, only the leading relativistic corrections $\sim\alpha^2$ were taken into account, and the following expression was derived for the decay probability of the $2p_{1/2}$ state:

$$\gamma_{\text{rel}} = W_0\left(1+\alpha^2\ln\frac{9}{8}\right), \qquad W_0 = 4\pi c\,\frac{2^8}{3^8}\,R_H\,\alpha^3. \tag{3}$$

For a further refinement of the value of γ, a systematic analysis has been carried out[7] of the leading radiative corrections $\sim\alpha^3$ and $\sim\alpha^3\ln\alpha$, i.e., the eigenenergy-shift and vacuum-polarization corrections to the $2p_{1/2}$–$1s_{1/2}$ transition amplitude.

The net contribution of these corrections to the transition probability is

$$\delta\gamma_{\text{rad}} = W_0\,\frac{32}{3\pi}\alpha^3\left[L\,(2{,}1)\,\frac{1}{8} - L\,(1{,}0) - \ln\frac{1}{\alpha^2} - \frac{1}{64} - \frac{19}{30}\right], \tag{4}$$

where $L\,(n,l)$ is the Bethe logarithm. As a result, combining (3) and (4), we find the following numerical value for the decay constant:

$$\begin{aligned} \gamma = \gamma_{\text{rel}} + \delta\gamma_{\text{rad}} &= 0{,}62648812\ (20)\cdot 10^{9}\ \text{s}^{-1}, \\ \tau &= 0.159619946\ (48)\cdot 10^{-8}\ \text{s}. \end{aligned} \tag{5}$$

In the calculation of γ we used the values of the fundamental constants α and R_H from Refs. 8 and 9. The relative error in γ is ~ 0.3 *ppm* and is determined by the error in the fine-structure constant. The finite size of the proton leads to a correction two orders of magnitude smaller.

Working from the experiments of Refs. 1 and 2 and the new value of the decay constant γ in (5), we then find the following values for the frequency,

$$\nu = 909{,}8934 \pm 0{,}0019\ \text{MHz}, \tag{6}$$

and the Lamb shift,

$$\delta(H, n=2) = 1067.8514 \pm 0.0019\ \text{MHz}. \tag{7}$$

The error in the theoretical value of γ is an order of magnitude smaller than the experimental error in (6) and (7).

We wish to emphasize again that the magnitude of the Lamb shift has been determined from the theoretical value of γ. The ratio of these quantities, ν/γ, is on the order of unity. However, it is a much simpler matter to calculate γ within $\lesssim 1$ *ppm* than to carry out a systematic quantum-electrodynamic calculation of ν with the same accuracy. The errors in the theoretical values presently available for the Lamb shift[5,6] are 10 *ppm* (~ 10 kHz).

The Lamb shift given by (7) turns out to be very close to the average value $\bar{\delta}(H, n=2) = 1057.8535$ MHz from the experiments of Refs. 3 and 4.

We note in conclusion that, in principle, this accuracy in the measurement of the Lamb shift (~ 2 kHz) makes it possible to extract the radius of the proton within an error of 0.007 fm from our experiment on the interference of atomic states. This accuracy is about twice as good as that of the most recent electron-scattering experiments.[10] On the other hand, this approach requires a theoretical quantum-electrodynamic calculation of the Lamb shift with the corresponding accuracy.

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