Mechanisms of spontaneous compactification of $N = 2$, $d = 10$ supergravitation

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It is shown that the supergravitation examined here has the possibility of spontaneous compactification, leading to $SU(3)\times SU(2)\times U(1)$ gauge fields in physical $d = 4$ space.

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Considerable attention is being given to the analysis of the mechanisms of spontaneous compactification of higher dimensinal spaces accompanying the interaction of Einstein fields with various types of gauge fields.[1] This interest is not accidental: It could be that it is precisely spontaneous compactification of the additional dimensions that gives rise to both the structure of internal symmetry groups and the hierarchy of their successive spontaneous breaking (playing the role of a Higgs mechanism), as well as to spontaneous supersymmetry breaking.

In this paper, we examine spontaneous sompactification of $N=2$, $d=10$ supergravitation. The physical fields in this variant are contained in three supermultiplet fields:

$$(l_M^{(M)},\ \psi_M^\sigma,\ A_{MN},\ \chi^\sigma,\ \Phi)\,(A_{MNL},\ \phi_M^\sigma)\ (A_M,\ \lambda^\sigma),$$

where $l_M^{(M)}$ is the gravitational field, ψ_M^σ, ϕ_M^σ are the Rarita-Schwinger fields, χ^σ,λ^σ are the spinor fields, A_{MN}, A_{MNL}, A_M are Abelian gauge fields, Φ is a scalar field [M, N, $L=0, 1, ..., 9$ are the world indices of a space with signature $(-, + ... +)$, (M) is the index of the tangent space, and $\sigma = 1, ..., 16$ is the spinor index in $d=10$].

The interaction of a gravitational field with gauge fields, present in each of the multiplets, can lead to spontaneous compactification of the 10-dimensional space. The mechanisms of spontaneous compactification, examined in Refs. 2 and 3, were based on the interaction of the field $l_M^{(M)}$ with the fields A_{MN} and A_{MNL}.

The presence of a third multiplet in $N=2$, $d=10$ supergravitation leads to additional possibilities, if the holonomy group of the compact space is a direct product of groups, one of which is Abelian. It should be emphasized that in contrast to the general situation,[4,5] compactification in spaces of this type can occur with the participation of only a vector Abelian field.

We shall examine the most interesting case, when the interaction of fields $l_M^{(M)}$ and A_M leads to spontaneous compactification of $d=10$ space in $adS^4\times CP(2)\times S^2$ [adS^4 is the anti-de-Sitter space, $CP(2)=[\mathrm{SU}(3)]/[\mathrm{SU}(2)\times U(1)]$, $S^2=\mathrm{SU}(2)/U(1)$].

The action for the interacting fields $l_M^{(M)}$ and A_M has the form

$$S=\int\Big(-\frac{E^{(10)}}{16\pi G}R^{(10)}-\frac{E^{(10)}}{4l^2}F_{MN}F^{MN}\Big)d^{10}x, \tag{1}$$

where $E^{(10)}=\det l_M^{(M)}$,$F_{MN}=\partial_M A_N-\partial_N A_M$, and G and L are the gravitational and gauge coupling constants.

The equations of motion following from (1) have the usual form

$$R_{MN}^{(10)}-\frac{1}{2}g_{MN}R^{(10)}=-\frac{8\pi G}{l^2}\Big(F_{ML}F_N^L-\frac{1}{4}g_{MN}F_{KL}F^{KL}\Big),$$
$$D_M F^M{}_N=0, \tag{2}$$

where D_M is the covariant derivative containing the metric connection, and $g_{MN}=l_M^{(M)}l_{N(M)}$ is the metric of the space $d=10$.

The vacuum solution of Eqs. (2), which leads to compactification of the tendemensional space in $adS^4\times CP(2)\times S^2$, is found from the condition[4]

$$D_M F_{NL} = 0, \tag{3}$$

When this condition is satisfied, the equations of motion for the gauge field A_M are satisfied identically. The imposition of this condition is due to the geometrical structure of the symmetric spaces $CP(2)$ and S^2, which are characterized by the covariant constancy of the curvature tensors, which in an orthogonal basis have the form

$$R^{(CP(2))}_{(a)(b)(c)}{}^{(d)} = -K_1\left(f_{(a)(b)}^{A}\, f_{A(c)}^{(d)} + f_{(a)(b)}^{8}\, f_{8(c)}^{(d)}\right),$$

$$R^{(S^2)}_{(i)(j)(k)}{}^{(l)} = -K_2\, C_{(i)(j)}^{3}\, C_{3(k)}^{(l)}\ , \tag{4}$$

where $K_1 > 0$, $K_2 > 0$ are the Gaussian curvatures of $CP(2)$ and S^2, which determine the characteristic dimensions of these spaces.

The structure constants $f_{(a)(b)}^{A}$, $f_{(a)(b)}^{8}$ of the group SU(3) in the Gell-Mann representation [$A = 1, 2, 3$: (a),(b) = 4, 5, 6, 7 are the indices of the space tangent to CP(2)] and the structure constants $C_{(i)(k)}^{3}$ of the group SU(2) [(i) and (k) are the indices of the space tangent to S^2] are normalized by the condition that the Killing-Cartan metrics of these groups correspond to the Kronecker symbols.

The components of the tensor of the field intensities F_{MN} which satisfy (3) have the following form in an orthogonal basis:

$$F_{(\mu)(\nu)} = 0,\ F_{(a),(b)} = \frac{l^2}{8\pi G}\,\mu_1 f^{8}_{(a),(b)},\ F_{(i)(k)} = \frac{l^2}{8\pi G}\,\mu_2 C^{3}_{(i)(k)}\ , \tag{5}$$

(μ) and (ν) are indices of the space tangent to adS^4, μ_1 and μ_2 are arbitrary numerical parameters.

From (2), (4), and (5) we can obtain the following values of K_1 and K_2 and the contracted curvature tensor for adS^4:

$$K_1 = \frac{1}{8}\,\frac{l^2}{8\pi G}\,(3\mu_1^2 - \mu_2^2),$$

$$K_2 = \frac{1}{8}\,\frac{l^2}{8\pi G}\,(7\mu_2^2 - \mu_1^2), \tag{6}$$

$$R^{(adS^4)} = \frac{1}{4}\,\frac{l^2}{8\pi G}\,(\mu_2^2 + \mu_1^2).$$

Since the metric of the 10-dimensional space contains only one time-like coordinate (i.e., $K_1 > 0$ and $K_2 > 0$), μ_1 and μ_2 must satisfy the inequality $7 > \mu_1^2/\mu_2^2 > 1/3$.

Perturbations of the vacuum state of the metric g_{MN}, which account for the fact that the gauge fields of the SU(3)×SU(2) group appear in the physical sector of the theory, are constructed in the standard manner[6] with the help of the Killing vectors of the spaces $CP(2)$ and S^2 and, in this case, in order to conserve the invariance relative to transformations of SU(3) and SU(2) groups [which are the groups of motions of $CP(2)$

amd S^2], depending on the coordinates of the 4-dimensional space time, the tensor $F_{MN}(x,y,z)$ is restructured and its components in an orthogonal basis assume the form

$$F_{(M)(N)} = \Big(F_{(\mu)(\nu)}(x) - \frac{8\mu_1}{3\mu_1^2-\mu_2^2} G^{\alpha}_{(\mu)(\nu)}(x)\, R^{8}_{\alpha}(y) - \frac{8\mu_2}{7\mu_2^2-\mu_1^2}\, H^{A}_{(\mu)(\nu)}(x)\, R^{3}_{A}(z);$$

$$\frac{l^2}{8\pi G}\mu_1 f^{8}_{(a)(b)};\ \frac{l^2}{8\pi G}\mu_2\, C^{3}_{(i)(k)}\Big), \tag{7}$$

where x, y, and z are the coordinates of the physical space and of the $CP(2)$ and S^2 spaces, respectively, $C^{\alpha}_{\mu\nu}(x)$ and $H^{A}_{\mu\nu}(x)$ are the tensors of the intensities of gauge fields of SU(3) and SU(2) groups ($\alpha = 1, 2, \ldots, 8$ and $A = 1, 2, 3$), and $R^{8}_{\alpha}(y), R^{3}_{A}(z)$ are elements of the transformation matrices of SU(3) and SU(2) groups in the adjoint representation.

From action (1), using relations (6) and (7) and taking into account the contribution of fluctuations above the vacuum state in the metric, after integrating over the coordinates of the additional dimensions, we obtain a Lagrangian that describes the system of interacting Einstein fields and gauge fields of SU(3)×SU(2)×U(1) groups in the physical $d = 4$ space

$$L = -\frac{E^{(4)}}{16\pi G^{(4)}}R^{(4)} - \frac{E^{(4)}}{4g_1^2}(G^{\alpha}_{\mu\nu})^2 - \frac{E^{(4)}}{4g_2^2}(H^{A}_{\mu\nu})^2 - \frac{E^{(4)}}{4(l^{(4)})^2}(F_{\mu\nu})^2 + E^{(4)}\Lambda . \tag{8}$$

The gravitational constant $G^{(4)}$ and the gauge interaction constants in the Lagrangian (8) are defined as follows in terms of starting constants G and l of the 10-dimensional action (1) and the volume $V_{(6)}$ of the compact space $CP(2)\times S^2$:

$$G^{(4)} = G/V_{(6)},\quad (l^{(4)})^2 = l^2/V_{(6)},\quad g_1^2 = (l^{(4)})^2\,\frac{(3\mu_1^2-\mu_2^2)^2}{2(7\mu_1^2-\mu_2^2)},$$

$$g_2^2 = (l^{(4)})^2\,\frac{3}{8}\,\frac{(7\mu_2^2-\mu_1^2)^2}{(15\mu_2^2-\mu_1^2)};$$

$\Lambda = \frac{1}{16}[(l^{(4)}/8\pi G^{(4)})]^2(\mu_1^2+\mu_2^2)$ is an anti-de-Sitter cosmological term.

As is evident from (8), spontaneous compactification of the additional dimensions in this mechanism, just as with a combination of this mechanism with all previously examined mechanisms,[2,3] is accompanied by the appearance of a cosmological term of the anti-de-Sitter type.

Since spontaneous supersymmetry breaking leads to the appearance of a cosmological term with opposite sign, the general solution of the problem concerning the magnitude of the cosmological constant requires an analysis of both boson and fermion excitations.

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