Values of dimensional quantities from Monte Carlo calculations in quantum chromodynamics

Yu. M. Makeenko and M. I. Polikarpov
Institute of Theoretical and Experimental Physics

(Submitted 18 July 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **38**, No. 8, 403–406 (25 October 1983)

An expression is derived for $\Lambda_L(\beta)$ to describe the behavior of the Monte Carlo data on the string tension coefficient in the transition region in the SU(3) lattice gauge theory. This expression leads to a 25% increase in $\Lambda_{\rm mom}$, while there are no changes in the other dimensional quantities (the deconfinement temperature, for example) found by the Monte Carlo method.

PACS numbers: 12.35.Cn, 11.15.Ha, 11.30.Jw

A nonperturbative calculation procedure using the Monte Carlo method which has recently been developed in quantum chromodynamics (QCD) can furnish the values of dimensional quantities such as the QCD parameter Λ, the temperature (T_c) at which quark confinement is violated, the hadron mass spectrum, etc. The criterion for determining whether the calculated values are pertinent to the continuous theory is their special dependence on the coupling constant g_0^2, dictated by the renormalizability of QCD. Specifically, all quantities with the dimensionality of mass which are calculated on a lattice with a step a are proportional to the parameter

$$\Lambda_L = \frac{1}{a}\left(\frac{16\pi^2}{11g_0^2}\right)^{\frac{51}{121}} \exp\left\{-\frac{8\pi^2}{11g_0^2}\right\} \tag{1}$$

 0021-3640/83/200489-05$01.00 © 1984 American Institute of Physics

(the polarization of the vacuum by quarks is ignored). To calculate the dimensional quantity we thus examine the exponential dependence of the Monte Carlo data on $1/g_0^2$ described by (1), and we then put the result in units of Λ_L.

To convert to physical units we must determine Λ_L, by equating one of the calculated dimensional quantities to its experimental value. It is customary to choose as this quantity the string tension coefficient K [experimentally, $K = (400\ \text{MeV})^2$], which was first calculated by the Monte Carlo method in calculations by Creutz[1] on a six-dimensional lattice.[4] The results of those and succeeding calculations (on lattices with a maximum dimensinality of $10^4)^2$ agree with the value $\Lambda_L/\sqrt{K} = (6 \pm 1) \times 10^{-3}$ (from which the value $\Lambda_L = 2$–3 MeV is found).

Actually, expression (1) is valid only in the limit $g_0^2 \to 0$. If g_0^2 is nonzero, a series in g_0^2 $(1 + c_1 g_0^2 ...)$ arises on the right side; the coefficients of this series are related to the third and succeeding coefficients of the series of the lattice perturbation theory for the Gell-Mann–Low function. Since $g_0^2 \approx 1$ is not small in practical Monte Carlo calculations with the SU(3) gauge group, we need to know how the results may change when this series is taken into account.

We have developed a method for approximately taking into account the contributions of the subsequent orders in the series in g_0^2 [ignored in expression (1)]. It turns out that these higher-order approximations increase Λ_L by about 25%. No changes are found in the absolute values (in MeV) of the other dimensional physical quantities.

To evaluate the contributions to Λ_L of the next orders in g_0^2 we consider the mixed lattice action

$$S = \sum_p \left\{ \frac{\beta}{3} \text{Re tr}\, U_p + \frac{\beta_A}{9} |\text{tr}\, U_p|^2 \right\}, \tag{2}$$

where U_p is the standard product of the variables $U_{x,\mu}$ along the boundary of the face p. In the limit $a \to 0$, action (2) becomes the action of a continuous theory with a coupling constant g_0^2:

$$6/g_0^2 = \beta + 2\beta_A - 5\beta_A/(\beta + 2\beta_A) + O((\beta + 2\beta_A)^{-2}). \tag{3}$$

The phase diagram on the β, β_A plane is known for action (2) from Monte Carlo calculations.[3] At $\beta_A > 1.2$ there is a line of first-order phase transitions in the phase plane, and expression (1) does not hold near this line. We reach this conclusion from the Monte Carlo calculations of Ref. 4, carried out for the SU(2) gauge group near the phase-transition point. It turns out that the results of these calculations can be described only by means of a refined expression,[5] which relates g_0^2 and Λ_L and takes the series in g_0^2 into account approximately.

For the SU(2) gauge group, however, the $\beta_A = 0$ axis, which corresponds to the Wilson action, with which most Monte Carlo calculations are carried out, lies quite far from the phase-transition line, and the series in g_0^2 may be ignored. In contrast, for the SU(3) gauge group the $\beta_A = 0$ axis lies much closer to the phase-transition line, and we would like to refine expression (1) by taking the series in g_0^2 into account approximately. This can be done by examining the mixed action (2), for which expression (1) becomes progressively more applicable as we move away from the phase-transition line. We thus select some $\beta_A < 0$ such that Λ_L in the mixed model is described well by expression (1) with the value given for g_0^2 by (3).

The expressions derived in Ref. 5, which are based on the property of lattice universality and which relate the values of the parameter Λ_L extracted for various values of β_A, can now be used to determine how, for the Wilson action, Λ_L should depend on β so that expression (1) will be exact for the mixed action at the given value of β_A. Proceeding in this manner, we find

$$\Lambda_L = \frac{1}{a}\left(\frac{16\pi^2}{11g^2}\right)^{\frac{51}{121}} \exp\left\{-\frac{8\pi^2}{11g^2} + \frac{10\pi^2}{9}\beta_A g^2\right\}, \tag{4}$$

where the refined coupling constant g^2 is given by

$$6/g^2 = \beta + \beta_A\left[2 - 2\omega(\beta) - \rho'(\beta)/\omega'(\beta)\right], \tag{5}$$

and the linear approximation in β_A is expressed exclusively in terms of

$$\omega(\beta) = \langle \frac{1}{3}\,\mathrm{tr}\,U_p \rangle; \qquad \rho(\beta) = \langle \,|\frac{1}{3}\mathrm{tr}\,U_p|^2 \rangle - \omega^2(\beta), \tag{6}$$

which are calculated for the Wilson action. Here the prime denotes the derivative with respect to β.

Figure 1 shows the results of a description of the Monte Carlo data of Ref. 6 for $\ln a^2K$. The dashed line is drawn for $\beta_A = 0$ and corresponds to the value[6] $\Lambda_L/\sqrt{K}$. The solid curve is constructed from expression (4) with $\beta_A = -0.7$ and the numerical values of ω and ρ; it corresponds to $\Lambda_L/\sqrt{K} = 10^{-2}$. We see that the solid curve correctly describes the deviation of the Monte Carlo data from a straight line which is observed at $\beta < 6$. We also generated corresponding descriptions for other values of $\beta_A < 0$. The values of the ratio $\Lambda_L/\sqrt{K}$ which give the best description of the Monte Carlo data for the given value of β_A are shown in Fig. 2. As β_A is increased, $\Lambda_L/\sqrt{K}$ initially increases by about 25% and then it becomes essentially constant over the interval $-0.5 > \beta_A > -1$; in other words, this ratio does not depend on which value of β_A is used. The observed decrease in $\Lambda_L/\sqrt{K}$ at $\beta_A > -1$ can be explained on the basis that expression (5), derived in the linear approximation in β_A, becomes inapplicable. The value $\Lambda_L/\sqrt{K} = 10^{-2}$ corresponds to that value of the QCD parameter Λ which corresponds to the moment regularization, $\Lambda_{\mathrm{mom}} = 84\Lambda_L \approx 340$ MeV. This value is quite different from that of Refs. 1 and 2: $\Lambda_{\mathrm{mom}} \approx 200$ MeV. Parisi *et al.*[7] have recently found evidence for a significant increase in Λ_{mom} (to ≈ 400 MeV) in the transition to large lattices. Our explanation of why the values found for Λ_{mom} for small lattices are too low is that these values are found with the help of the simple expression (1), which does not describe the actual β dependence of theMonte Carlo data at $\beta < 6$.

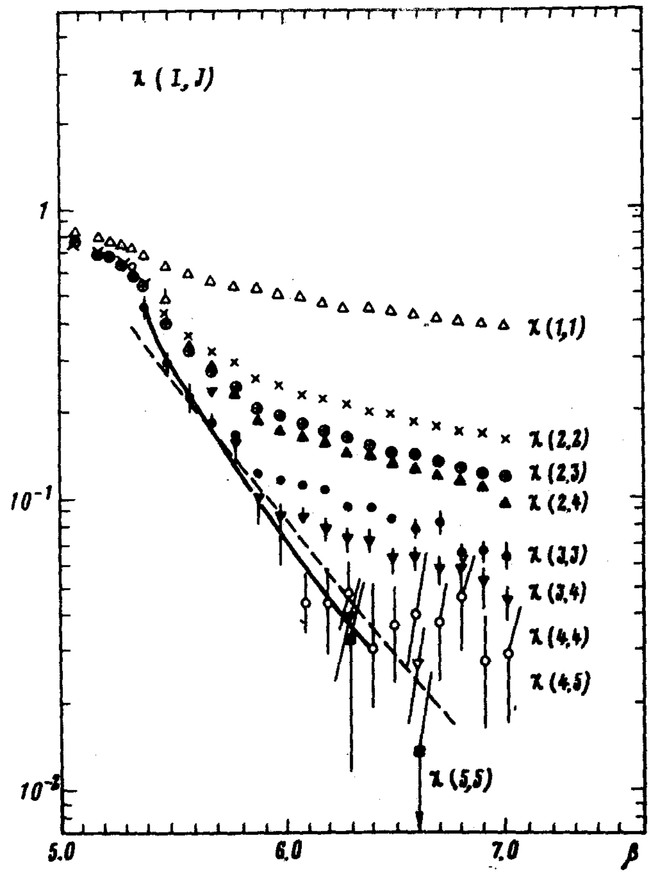

FIG. 1.

Expression (1) has been used[8] to derive the following deconfinement temperature: $T_c = 76\Lambda_L \approx \Lambda_{\rm mom}$. On the basis of this result, it was asserted in Ref. 7 that T_c increases to 370 MeV, since $\Lambda_{\rm mom}$ increases. This value of T_c is very high, and it cannot be attained by the heavy-ion accelerators presently under construction. We

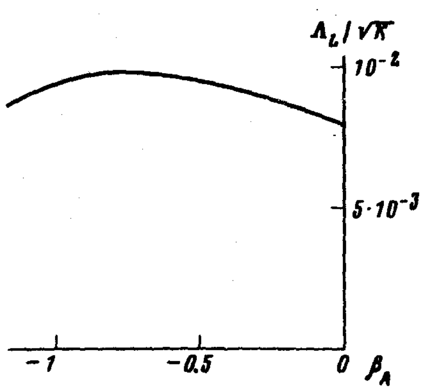

FIG. 2.

have analyzed the Monte Carlo data on T_c with the help of expression (4), and we found a lower value of T_c in units of Λ_L: $T_c = 60\Lambda_L$. This value corresponds to $T_c \approx 240$ MeV, which is substantially lower and not as different from the value $T_c \approx 200$ MeV given in Ref. 8. We expect that this property will remain valid for other dimensional quantities calculated by the Monte Carlo method.

We wish to thank R. Petronzio for prepublication information about Ref. 7. We also thank J. Kogut, N. Christ, M. Creutz, and A. Muller for a discussion during the Soviet-American Conference on Gauge Theories at Erevan.

[1]M. Creutz, Phys. Rev. Lett. **45**, 313 (1980).
[2]E. Pietarinen, Nucl. Phys. **B190**, [FS3], 349 (1981).
[3]G. Bhanot, Phys. Lett. **108B**, 337 (1982).
[4]G. Bhanot and R. Dashen, Phys. Lett. **113B**, 299 (1982).
[5]Yu. M. Makeenko, M. I. Polikarpov, and A. V. Zhelonkin, Phys. Lett. **126B**, 82 (1983).
[6]M. Fukugita, T. Kaneko, and A. Ukawa, Tokyo Preprint KEK-TH63, 1983.
[7]G. Parisi, R. Petronzio, and F. Rapuano, CERN Preprint TH. 3596, 1983.
[8]K. Kajantie, C. Montonen, and E. Pietarinen, Z. Phys. **C9**, 253 (1981); J. Kogut *et al.*, Phys. Rev. Lett. **50**, 393 (1983).

Translated by Dave Parsons
Edited by S. J. Amoretty