

Stochastic instability of the motion of particles in free-electron lasers

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Under the optimum working conditions of a free-electron laser, a stochastic instability occurs in the motion of the beam particles. The transition from regular to stochastic motion is a continuous one.

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In this letter we show that a stochastic instability occurs in the motion of the beam particles in a free-electron laser operated under optimum conditions. This instability gives the particles an energy spread and correspondingly reduces the efficiency at which the energy of the relativistic electron beam is converted into electromagnetic radiation. In addition, the nature of the emission may change: The coherent emission may give way to a complex, irregular emission.

Let us consider the motion of beam particles in the pump wave $A_1 \sin \phi_1$ and in the field of the scattered electromagnetic wave, which we represent as the sum of the longitudinal resonator modes: $A = \sum_s A_s \sin \phi_s$, where A_m is the vector potential of the field, and $\phi_m = k_m z - \omega_m t$. The beam is assumed to be directed along the z axis, and the wave vectors of the wave are assumed to be parallel to the beam velocity. The equations of motion of the beam electrons can then be written

$$\frac{d\epsilon}{dt} = -\frac{1}{(1-\epsilon)} \frac{\partial}{\partial t} \left(\sum_{m=1, s} a_m \sin \phi_m \right)^2, \quad (1)$$

$$\frac{dz}{dt} = c \left\{ 1 - (1-\epsilon)^{-2} \left[\gamma_0^{-2} + 2 \left(\sum_{m=1, s} a_m \sin \phi_m \right)^2 \right] \right\}^{1/2},$$

where

$$\epsilon \equiv 1 - \gamma/\gamma_0; \quad a_m \equiv eA_m / (\sqrt{2} mc^2 \gamma_0); \quad \gamma^{-2} = 1 - v_0^2/c^2.$$

We assume that the oscillation velocity of the particles and the change in their energy are quite small ($a_m \ll 1$, $\epsilon \ll 1$). Adopting the coordinate z as an independent variable, we can rewrite Eqs. (1) as

$$\frac{d\epsilon}{dz} = \frac{a_1}{v_0} \sum_s a_s \omega_{s1} \sin \theta_{s1}; \quad \frac{d\theta_{s1}}{dz} = k_{s1} - \frac{\omega_{s1}}{v_0} - \frac{\omega_{s1} \epsilon}{v_0 \beta_0^2 \gamma_0^2}; \quad (2)$$

where $k_{s1} \equiv k_s + k_1$, $\omega_{s1} = \omega_s + \omega_1$, and $\theta_{s1} = \phi_s + \phi_1$.

We assume that a particle is initially at resonance with the sum wave with the phase θ_{s1} . The frequency of the scattered wave is higher than that of the pump wave in this case if the pump wave propagates along the beam with a phase velocity lower than the beam velocity.^{1,2} For the scattering of an electromagnetic wave which is propagating opposite the beam in a vacuum, we must replace ω_1 by $-\omega_1$ in Eqs. (2) and in the equations below; in the case of a magnetostatic pump, we must set $\omega_1 = 0$.

The beam particles interact most efficiently with the field under the resonant condition $\theta_{s1} = \text{const}$. If the wave amplitudes are small, we may assume that this interaction involves only a single resonance, and we may ignore the others. In this case the motion of a particle is described by the equation of a mathematical pendulum, from which we easily find the width of the nonlinear resonance to be $\delta\kappa = (\omega_{s1}/v_0)[(a_1 a_s)^{1/2}/\beta_0 \gamma_0]$. The mathematical-pendulum approximation is valid if the distance between resonances, $\Delta\kappa$, equal to $\Delta k_s |1 - [(1/v_0)(d\omega_s/dk_s)]|$, is much greater than the width of the nonlinear resonance, $\delta\kappa$. If, on the other hand, $\delta\kappa > \Delta\kappa$, then the nonlinear resonances will overlap, and the motion of the particles will become random.³ The scattered-wave amplitude a_{cr} , at which the nonlinear resonances begin to overlap ($\Delta\kappa = \delta\kappa$), is given by

$$a_{cr} = (v_0 \beta_0 \gamma_0 \Delta\kappa / \omega_{s1})^2 / a_1.$$

The ratio of a_{cr} to the optimum scattered-wave amplitude—that which maximizes the efficiency—is given by the following expressions for the cases of an oscillator⁴ and an amplifier^{4,2}:

$$\frac{a_{cr}}{a_{opt}} = \begin{cases} 0,25 \gamma^{-4} & (\text{oscillator}) \\ \left(\frac{\Delta k_s}{2\gamma_0^2 \delta k_s} \right)^2 & (\text{amplifier}) \end{cases}, \quad (3)$$

where $\delta k_s = k_s \left(\frac{a_1^2 \omega_{b1}^2}{4\gamma_0^2 \omega_{s1} \omega_2} \right)^{1/3}$ is the factor by which the scattered wave is amplified in the amplifier; $\Delta k_s = k_{s+1} - k_s$ and $\omega_{b1}^2 = 4\pi e^2 n_b / (m\gamma_0)$.

It can be seen from expression (3) that the stochastic instability sets in at a field well below the optimum field.

The stochastic instability may prove unimportant if the dimensions (L) of the interaction region (the length of the electrodynamic structure) are much smaller than the distance l_k over which the phase correlation is lost upon the onset of the stochastic instability. Determining l_k from

$$l_k = [\delta k \ln(\frac{\delta k}{\Delta k})]^{-1},$$

we find the following expression for the extreme scattered-wave amplitude (a_{ex}) required for loss of correlation:

$$a_{ex} = x^2 a_{cr}, \quad (4)$$

where x is the solution of the equation $x \ln x = 2\gamma_0^2 / \pi$. From this expression we find an estimate of the upper limit of the ratio of a_{ex} to the optimum amplitude of the excited waves:

$$\frac{a_{ex}}{a_{cr}} < \begin{cases} \pi^{-2} & (\text{oscillator}) \\ (L \delta k_s)^{-2} & (\text{amplifier}) \end{cases}$$

The loss of correlation in the motion of the particles in an oscillator thus occurs at amplitudes below the optimum amplitudes.

One way to suppress the stochastic instability is to arrange a nonlinear suppression of spatial modes by a single, preferred mode. At low beam currents, this is the way in which a free-electron laser operates.⁵ As the current is raised, however, the fundamental mode becomes unstable, and other modes are excited.

To estimate the number (ΔN) of particles in random motion upon the excitation of the other modes, while their amplitudes are still small, we find the width of the stochastic layer. From (2) we find the expression

$$\frac{d^2 \theta_{s1}}{d\xi^2} = \sin \theta_{s1} + \alpha \sin \left(\frac{\omega'_{s1}}{\omega_{s1}} \theta_{s1} + \frac{\omega'_{s1}}{\Omega} \right),$$

where $\xi \equiv \delta k z$; $\alpha \equiv (a'_s / a_s)(\omega'_{s1} / \omega_{s1}) \ll 1$; $\Omega \equiv \delta k [(k_{s1} / \omega_{s1}) - (k'_{s1} / \omega_{s1})]^{-1}$; and the prime denotes a property of the other modes. The energy width of the stochastic layer near the separatrix can be estimated from the following inequality, according to Ref. 3:

$$\Delta H = (1 - H_0) \lesssim \frac{4\alpha}{\pi} \frac{\omega'_{s1}}{\Omega} \exp\left(-\pi \frac{\omega'_{s1}}{\Omega}\right),$$

where

$$H_0 \equiv [(d\theta_{s1}/d\xi)^2/2 + \cos\theta_{s1}]|_{\xi=0}; \quad \Delta N \sim \frac{\Delta H}{H_0} N.$$

We can thus draw the following picture of the onset of the stochastic instability in a free-electron laser. At low beam currents, at which only a single spatial mode is excited, the particle motion and the field characteristics are regular. As the current is increased, other modes are excited, and a small group of particles in stochastic motion appears near the separatrix. A further increase in the current increases the number of these particles, and when the amplitudes of the other modes reach levels satisfying condition (4) the stochastic instability extends to all the beam particles. The degree of randomization of the particle motion in a free-electron laser thus increases continuously. We might note that this road to chaos is apparently characteristic of many other systems with electron beams.

These results show that the amplitudes of the excited fields must be well below the optimum amplitudes in oscillators. In amplifiers, on the other hand, the stochastic instability occurs only over distances greater than the reciprocal of the gain factor. Since the field in an amplifier increases along the coordinate, reaching the value required for an overlap of resonances only at a distance on the order of $(\delta k_s)^{-1}$, we may expect that the stochastic instability will not be important for amplifiers. However, the change in the amplitude along the coordinate may cause the stochastic instability to occur at lower amplitudes of the scattered wave.

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¹V. A. Buts, V. I. Miroshnichenko, and V. V. Ognivenko, Zh. Tekh. Fiz. **50**, 2257 (1980) [Sov. Phys. Tech. Phys. **25**, 1320 (1980)].

²V. V. Ognivenko, Radiotekh. Elektron. **27**, 1818 (1982).

³G. M. Zaslavskii and N. N. Filonenko, Zh. Eksp. Teor. Fiz. **54**, 1590 (1968).

⁴V. L. Bratman, N. S. Ginzburg, and M. I. Petelin, Zh. Eksp. Teor. Fiz. **76**, 930 (1979) [Sov. Phys. JETP **49**, 469 (1979)].

⁵N. S. Ginzburg and S. P. Kuznetsov, in: Relyativistskaya vysokochastotnaya elektronika (Relativistic Radio-Frequency Electronics), IPF Akad. Nauk SSSR, Gor'kii, 1981, p. 101.

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