Test of CPT symmetry in the decays of neutral kaons

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The Bell-Steinberger unitarity relation is used along with the experimental data available to calculate the parameter of the *CPT* violation, $\tilde{\Delta}$, and that of the T violation, ϵ, in the decays of K^0 mesons. It is shown that if the *CP* violation in three-pion decays of K^0 mesons is not greater than in two-pion decays, then the parameters $\mathrm{Re}\tilde{\Delta}$ and $\mathrm{Im}\tilde{\Delta}$ differ from zero by two standard deviations.

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The CPT and T invariances in the decays of K^0 mesons can be tested experimentally by making use of the Bell-Steinberger unitarity relation.[1] Until recently, however, we lacked that statistically adequate data base on the parameter η_{000}, which determines the *CP*-invariant decay $K_s \to 3\pi^0$, which we would need for a thorough comparison of this relation with experiment. This parameter has now been determined[2] from 632 $K^0 \to 3\pi^0$ decay events: $\eta_{000} = A(K_S \to 3\pi^0)/A(K_L \to 3\pi^0) = (-0.08 \pm 0.18) + i(-0.05 \pm 0.27)$. Here the A's are the decay amplitudes. Using this result along with other data available on the decays of K^0 mesons, we can work from the unitarity relation without appealing to any indirect arguments about the parameter η_{000}, as has been the approach in the past.[3,4]

Following Shubert,[3] we write the Bell-Steinberger relation in the form

$$(1 + i\mu)(\mathrm{Re}\epsilon - i\mathrm{Im}\tilde{\Delta}) = \epsilon_0 + \alpha . \tag{1}$$

Here $\mu = 2(m_L - m_S)/(\Gamma_S + \Gamma_L)\hbar$, where $m_{L,S}$ are the masses of the K_L and K_S mesons, and $\Gamma_{L,S}$ are their decay widths. The other quantities in (1) are complex. We are using the following parameters:

$$\epsilon_0 = A(K_L \to 2\pi,\ I = 0) / A(K_S \to 2\pi,\ I = 0),$$
$$\alpha = (\Gamma_S + \Gamma_L)^{-1} \sum_j A(K_S \to j)^* A(K_L \to j).$$

In these expressions, I is the isospin, and $j = \pi^+\pi^-\pi^0,\ 3\pi^0,\ 2\pi(I=2),\ \pi^\pm e^\mp \nu,\ \pi^\pm \mu^\mp \nu$.

The known[3,5] parameters ϵ and $\tilde{\Delta}$ are related to ϵ_0 by

$$\epsilon_0 = \epsilon - \tilde{\Delta} . \tag{2}$$

For ϵ_0 and α we have the expressions

$$\epsilon_0 = [\, 2\eta_{+-}(1 + \omega) + \eta_{00}(1 - 2\omega)\,]/3 . \tag{3}$$

$$\alpha = (\Gamma_S + \Gamma_L)^{-1}\{\Gamma_L(\pi^+\pi^-\pi^0)\eta^*_{+-0} + \Gamma_L(3\pi^0)\eta^*_{000} + 2(\Gamma_S + \Gamma_L)\omega^*\epsilon_2$$
$$+ [\Gamma_L(\pi e \nu) + \Gamma_L(\pi\mu\nu)][\delta(1 + 2\mathrm{Re}x) - 2i\mathrm{Im}x]\} , \tag{4}$$

where the η's are the ratios of the amplitudes of the CP-forbidden decays of K_L and K_S mesons to the amplitudes of the corresponding CP-allowed decays;

$$\epsilon_2 = (1/\sqrt{2})\, A(K_L \to 2\pi, I=2)\, /\, A\,(K_S \to 2\pi,\ I=0),$$

$$\omega = (1/\sqrt{2}) A_2(K_S \to 2\pi,\ I=2)/\, A_0(K_S \to 2\pi,\ I=0),$$

and

$$\delta = [\Gamma_L(\pi^- l^+ \nu) - \Gamma_L(\pi^+ l^- \tilde{\nu})] / [\Gamma_L(\pi^- l^+ \nu) + \Gamma_L(\pi^+ l^- \tilde{\nu})]\ ,$$

$$x = A(\widetilde{K}^0 \to \pi^- l^+ \nu) / A\,(K^0 \to \pi^- l^+ \nu);$$

and[3)]

$$\epsilon_2 = [\,\eta_{+-}(1+\ \omega) - \eta_{00}(1-2\omega)]/3$$

$$\omega = (1/\sqrt{2})\mathrm{Re}(A_2/A_0)\exp[\,i(\delta_2 - \delta_0)\,{}^{1)}.$$

In the last expression, δ_2 and δ_0 are the $\pi\pi$-scattering phase shifts in the states with isospins 2 and 0.

In calculating the experimental values of μ, ϵ_0, and α, we used the tabulated data from Ref. 6, the world-average value[7] $\eta_{+-0} = (0.05 \pm 0.07) + i(0.26 \pm 0.13)$, the value[2] given above for η_{000}, and the parameter value[6] $x = (0.009 \pm 0.020) + i(-0.004 \pm 0.026)$. To determine ω we have $\delta_2 - \delta_0 = (-45.3 \pm 4.6)^\circ$ (Ref. 8) and $\mathrm{Re}\omega = 0.018 \pm 0.002$. The latter value was calculated from equations taken from Ref. 9. As a result, we have

$$\mu = 0{,}953 \pm 0{,}005, \tag{5}$$

$$\epsilon_0 = [(1{,}535 \pm 0{,}063) + i\ (1{,}686 \pm 0{,}052)] \times 10^{-3}, \tag{6}$$

$$\alpha = [(-0.006 \pm 0{,}068) + i\ (-0{,}026 \pm 0{,}120)] \times 10^{-3}. \tag{7}$$

Figure 1 shows the contributions of the various decays to the parameter α.

Solving system (1), (2), and using the values in (5)–(7), we find

$$\epsilon = [(1{.}62 \pm 0{,}05) + i\ (1{,}58 \pm 0{,}09)] \times 10^{-3}, \tag{8}$$

$$\widetilde{\Delta} = [(0{,}10 \pm 0{,}07) + i\,(-0{,}11 \pm 0{,}10)] \times 10^{-3}. \tag{9}$$

In calculating the errors in (8) and (9) we took into account the correlations between the real and imaginary parts of the parameter ϵ_0 and α, respectively. The correlation coefficients ρ were determined by the Monte Carlo method:

$$\rho(\mathrm{Re}\,\epsilon_0, \mathrm{Im}\,\epsilon_0) = -\ 0{,}68\ \text{и}\ \ \rho\,(\mathrm{Re}\alpha, \mathrm{Im}\,\alpha) = -\ 0{,}64.$$

It follows from result (8) that $\epsilon \neq 0$; i.e., T invariance is violated in the decays of K^0 mesons. This conclusion has been reached previously, but now the parameter ϵ has been determined more accurately than in the earlier work.[3,5]

We turn now to the quantity $\widetilde{\Delta}$. We should not only take the value in (9) into account but also adopt the extremely safe assumption that the CP violation in the

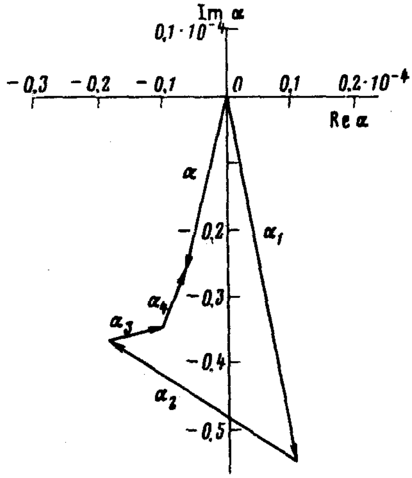

FIG. 1. $\alpha_1 - \pi^+\pi^-\pi^0$, $\alpha_2 - 3\pi^0$, $\alpha_3 - 2\pi$, $I = 2$, $\alpha_4 - \pi^- l^+ \nu$.

three-pion decays of K^0 mesons is no greater than that in two-pion decays; i.e., we should set $\eta_{+-0} = \eta_{000} = \eta_{+-}$. Using expressions (3) and (4) and the values in (6) and (7), we then easily find the estimate $|\alpha| \leqslant 5\times 10^{-3}|\epsilon| \cong 10^{-5}$. Furthermore, as is shown by an analysis of the solutions of system (1), (2) for the parameter $|\tilde{\Delta}|$, the value of $|\tilde{\Delta}|$ varies only very slightly over the interval $0 \leqslant |\alpha| \lesssim 10^{-5}$, remaining constant within better than 5%. For calculations of $\tilde{\Delta}$ in this region we can therefore accurately set $\alpha = 0$. Solving system (1), (2) under this condition, we then find

$$\tilde{\Delta}(a = 0) = [(0.11 \pm 0.05) + i\,(-0.12 \pm 0.05)] \times 10^{-3}.$$

In other words, we find a difference of two standard deviations from zero values for the parameters $\mathrm{Re}\tilde{\Delta}$ and $\mathrm{Im}\tilde{\Delta}$.

The condition $\alpha = 0$ is evidently equivalent to ignoring all pathways other than $K^0 \to 2\pi$ in the unitarity relation. We might note that of all the parameters characterizing these decays that which has been measured least accurately is the phase shift $\phi_{00} = \arg\eta_{00}$. The experimental value of this phase shift differs by two standard deviations from the prediction of the superweak-interaction model, which is

$$\varphi_{00} = \varphi_{+-} = \varphi_{SW} = \mathrm{arc\,tg}\,[2(m_L - m_S)/(\Gamma_S - \Gamma_L)\hbar] = (43.72 \pm 0.14)^0 .$$

The measured value of the phase shift ϕ_{+-}, on the other hand, is approximately equal to φ_{SW}:$\varphi_{+-} = (44.6 \pm 1.2)^0$. An approximate equality $\varphi_{00} - \varphi_{+-} \cong 0$ can also be found in other models for the violation of *CP* invariance, in particular, the Kobayashi-Mascawa model (see Ref. 10, for example). It thus appears quite likely that the experimental phase shift $\phi_{00} = (54 \pm 5)^\circ$ (Ref. 6) is responsible for the deviation of $\tilde{\Delta}$ from zero at $\alpha = 0$.

We can also determine how the parameters of the K^0 decays, which appear in the unitarity relation, must change in order to satisfy the *CPT* theorem. To resolve this question, we use the method of least squares to fit the available experimental values of the parameters φ_{00}, φ_{+-}, $\mathrm{Im}\eta_{000}$, $\mathrm{Im}\eta_{+-0}$, $\mathrm{Re}x$, and $\mathrm{Im}x$ to the condition $\tilde{\Delta} = 0$. As a result of this fitting procedure, we find $\phi_{00} = (48.7 \pm 3.7)^\circ$ and $\phi_{+-} = (44.0 \pm 1.1)^\circ$, i.e., a difference $\phi_{00} - \phi_{+-} = (4.7 \pm 3.8)^\circ$. Working from the original values we would have[6] $\phi_{00} - \phi_{+-} = (9.4 \pm 5.1)^\circ$. A corresponding fitting for $\alpha = 0$ yields $\phi_{00} = (44.1 \pm 2.2)^\circ$ and $\phi_{+-} = (43.4 \pm 1.1)^\circ$, i.e., a difference $\phi_{00} - \phi_{+-} = (0.7 \pm 2.5)^\circ$, in approximate agreement with the theoretical predictions.

The calculated results on the *CPT*-violation parameter $\tilde{\Delta}$ found in this paper do not support the earlier conclusion that the experimental data on the decays of K^0 mesons are in complete agreement with the *CPT* theorem.[3,5] This conclusion was reached by Shubert *et al.*[3] because of the inadequate experimental data available at that time. The compatibility of the calculated value of $\tilde{\Delta}$ with zero in Cronin's review[5] was the result of a great deal of uncertainty regarding the parameter α, attributable primarily to the absence of accurate experimental limitations on the contributions of the $K_S \to 3\pi$ pathways.

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3)In choosing the parameter ω in this form we are ignoring a possible rotation of the ω phase because of hypothetical *CPT*-forbidden but *CP*-allowed amplitudes. This possibility would have to be taken into account only if these amplitudes were very large—on the order of the *CPT*- and *CP*-allowed amplitudes. This and certain other questions (in particular, a possible *CPT* violation in the amplitude ϵ_2) will be discussed in detail in a separate paper.

1J. S. Bell and J. Steinberger, Proceedings of the Oxford International Conference on Elementary Particles, 1965, p. 195.
2V. V. Barmin *et al.*, Twenty-first International Conference on High Energy Physics, Paris, 1982; Phys. Lett. **128B**, 129–132.
3K. R. Shubert *et al.*, Phys. Lett. **31B**, 662 (1970).
4S. Gjesdal *et al.*, Phys. Lett. **52B**, 119 (1974).
5J. W. Cronin, Rev. Mod. Phys. **53**, 373 (1981).
6Particle Data Group, Phys. Lett. **111B**, (1982).
7M. Baldo-Ceolin *et al.*, Nuovo Cim. **25A**, 688 (1975).
8T. J. Devlin and J. O. Dickey, Rev. Mod. Phys. **51**, 237 (1979).
9L. B. Okun', Leptony i kvarki (Leptons and Quarks), Moscow, 1981, p. 77.
10M. B. Voloshin, Prepring No. 22, Institute of Theoretical and Experimental Physics, Moscow, 1981.

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