Collective self-effect of sound in a liquid with gas bubbles

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A new mechanism for a self-effect of sound in a liquid with gas bubbles is examined. The effect results from an average motion of the gas bubbles over the period of the sound wave and from the coalescence of the bubbles under the influence of the radiation-pressure force and the hydrodynamic interaction of the bubbles in the acoustic field.

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A liquid with gas bubbles is a medium with extremely complex nonlinear relaxation properties, playing a role in acoustics which might be compared with the role of a plasma or a resonant laser medium in electrodynamics. A variety of self-effects can occur in such a liquid, e.g., a self-focusing of sound.[1,2] This effect occurs by a cavitation mechanism: the nucleation of bubbles in the liquid by intense ultrasound. The possibility of a self-effect of sound by virtue of the cubic nonlinearity of a single bubble has also been studied.[3] This mechanism again requires rather strong acoustic fields.

In the present letter we wish to propose a more efficient mechanism (i.e., one for which the threshold sound intensity is lower) for a self-effect of sound. This mechanism involves a spatial redistribution of the bubbles caused by the average radiation-pressure force and the average force of the interaction between the bubbles. We will discuss some new effects in acoustics that stem from this mechanism.

The collective behavior of bubbles in an intense sound wave can be described by means of hypotheses similar to those used in plasma physics: The effect of the external field on an individual bubble can be described by introducing an average macroscopic field and by ignoring the incoherent component of this field. We also assume that the oscillations of an individual bubble are linear. The behavior of the quasiharmonic field, with the potential $\varphi = \frac{1}{2}[\Psi(\mathbf{r},\mu t)\exp(i\omega t) + \text{c.c.}]$, where Ψ is a complex amplitude that varies slowly over time, is then described by[4]

$$\Delta\Psi + \left[k^2 - \int_0^\infty (q + i\gamma) N(R, \mathbf{r}, \mu t) dR\right]\Psi = 0. \tag{1}$$

Here $k = \omega/c$, c is the sound velocity in the pure liquid, N is the bubble distribution in the radius R,

$$\gamma = 4\pi R\delta[(1 - S^2)^2 + \delta^2]^{-1}, \quad q = 4\pi R(1 - S^2)[(1 - S^2)^2 + \delta^2]^{-1}, \quad S = \omega_0/\omega,$$

and ω_0 and δ are the resonant frequency and damping rate of the natural oscillations of the bubble. The term of order Ψ_t can be ignored for the processes which we will be discussing below.

In the linear approximation it would be sufficient to consider the monopole oscillations of the bubbles, but the nonlinear self-effects with which we are concerned here are related in a fundamental way to the dipole oscillations, which given rise to an average motion of the bubbles in space. The velocity of this motion, **U**, is[5]

$$\mathbf{U} = \mathbf{U}_0 - \frac{q}{24\pi R\nu}(\Psi\nabla\Psi^* + \Psi^*\nabla\Psi) + \frac{i\gamma}{24\pi R\nu}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*), \tag{2}$$

where $\mathbf{U}_0$ is the bubble velocity in the absence of the sound field, and ν is the viscosity of the liquid.

We can find an equation for the bubble distribution function $N(R,\mathbf{r},\mu t)$ from the balance equation for the number of bubbles in an element of the phase space, taking into account the movements of bubbles out of this element or their appearance in it as a result of a possible coalescence:

$$N_t + \mathrm{div}(N\mathbf{U}) = -N\int_0^\infty N(R')\sigma(R,R')|\mathbf{U}(R) - \mathbf{U}(R')|\,dR'$$

$$+ \int_0^R N(R'')N(R')\sigma(R',R'')|\mathbf{U}(R'') - \mathbf{U}(R')|\,dR'. \tag{3}$$

Here σ is the collision cross section, and according to Ref. 5 we have $\sigma|\mathbf{U}_1 - \mathbf{U}_2| = 4\pi\kappa$ for $\kappa \geqslant 0$ and 0 for $\kappa \geqslant 0$, where

$$\kappa = \frac{R_1 + R_2}{3\nu}\,\frac{(S_1^2 - 1)(S_2^2 - 1) + \delta_1\delta_2}{[(S_1^2 - 1)^2 + \delta_1^2][(S_2^2 - 1)^2 + \delta_2^2]}\,|\Psi|^2 .$$

We treat the specific effects in the one-dimensional formulation, ignoring the effect of the bubbles that appear during coalescence, and we assume that we initially have bubbles of only a single radius. Equations (1) and (3) then become

$$\Psi_{xx} + [K^2 - (q + i\gamma)N]\Psi = 0, \tag{4}$$

$$N_t + (NU)_x = -2\gamma|\Psi|^2 N^2/3\delta\nu. \tag{5}$$

Analysis of systems (2), (4), and (5) reveals, in particular, the following effects:

1. Self-modulation of a traveling wave. In the field of a traveling harmonic sound wave (which is generally damped) there can be an equilibrium state with a constant bubble concentration N_0. This state, however, is not stable. Seeking a solution of the original equations in the form $\Psi = [\Psi_0 + \Psi_1(x,t)]\exp(-i\eta x)$, $N = N_0 + n(x,t)$, where $\eta = [K^2 - (q + i\gamma)N_0]^{1/2}$, and Ψ_1 and n are small perturbations, we find a linearized system of equations for Ψ_1 and n. For solutions of the type $\exp[i(kx - \Omega t)]$ we find from this system the dispersion relation

$$\Omega = 2i\beta[1 + (4|\eta|^2 + 2i|\,\mathrm{Im}\,\eta|k)/(k^2 - 4|\eta|^2 - 4i|\,\mathrm{Im}\,\eta\,|k)^{-1}], \tag{6}$$

where $\beta = q^2 N_0^2|\Psi_0|^2/24\pi\nu R$. It follows that we have $\mathrm{Im}\Omega > 0$ for all k which satisfy the condition $|k| \geqslant 2|\eta|$. The gross rate $\mathrm{Im}\Omega$ reaches its maximum value in this region at $|k| \simeq 2|\eta|$; this maximum value is $|\eta|\beta/2\,\mathrm{Im}\eta$. A homogeneous bubble distribution is thus unstable with respect to spatial modulation (an exception is the case of resonant bubbles, and $\beta = 0$). The physical meaning of this instability is easy to see: When inhomogeneities of the density arise, a back-scattered wave appears, and a standing

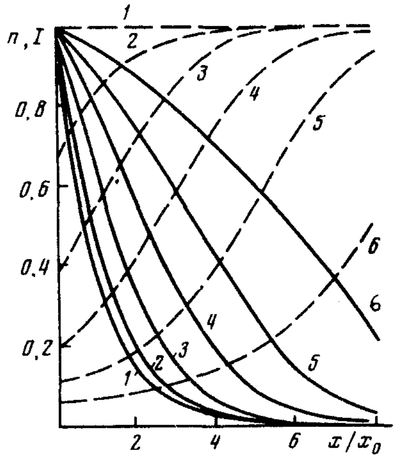

FIG. 1. The concentration $n = N/N_0$ (dashed curves) and the sound intensity $I = F/F_0$ (solid curves) versus the distance at various times. 1—$t = 0$; 2—0.5 t_0; 3—1.7 t_0; 4—4 t_0; 5—8 t_0; 6—16 t_0.

component appears in the field. Bubbles become grouped in this standing component, etc. We thus have a self-induced scattering due to a concentration mechanism, which drives an instability. In the pulsed case this mechanism would give rise to an acoustic echo, and in the three-dimensional case it would cause a phase conjugation of the sound wave (this effect has heretofore been attributed to other mechanisms[6]).

2. Self-induced transmission of the sound. We assume that a medium with an initial bubble concentration N_0 is produced in the $x = 0$ plane by a traveling sound wave with an amplitude Ψ_0. Ignoring the scatter field, we easily find from (2), (4), and (5) the balance equations

$$\Phi_x + \frac{\gamma N}{K}\Phi = 0, \tag{7}$$

$$N_t + \frac{\partial}{\partial x}\left[N\left(U_0 - \frac{q}{24\pi R\nu}\Phi_x + \frac{\gamma K}{24\pi R\nu}\Phi\right)\right] + \frac{2\gamma}{3\delta\nu}N^2\Phi = 0, \tag{8}$$

where $\Phi = |\Psi|^2$. First, ignoring the radiation pressure exerted on the bubbles [the second term in (8)], we find an implicit analytic solution of Eqs. (7) and (8):

$$x/x_0 = Ei[\ln(1 + t/t_0)] - Ei[\ln(N_0/N)], \tag{9}$$

where $x_0 = K/\gamma N_0$, $t_0 = 3\delta\nu/2\gamma\Phi_0 N_0$, and Ei is the integral exponential function.

The result of this solution is shown in Fig. 1 as the dependence of the field intensity and the bubble concentration on the distance for various values of t. We see that the damping of the wave decreases over time because of the coalescence of bubbles. The scale time for the self-induced transmission is on the order of t_0.

3. Envelope shock waves. The radiation pressure creates a situation in which field intensity jumps can move in a steady state and can completely drive bubbles out of a given phase volume. Such stationary waves can be found both from Eqs. (7) and (8)

(i.e., by ignoring the scatter field) and from the general equations (2), (4), and (5). The velocity of this wave is $V = U_0 + K\delta\Phi_0/3\nu[(1 - S^2)^2 + \delta^2]$, where Φ_0 is the final value of the sound amplitude. In the case of resonant bubbles, we find a steady-state solution of the following type from (7) and (8): $\Phi = \Phi_0(1 + \exp z)^{-1}$; $N = N_0[1 + \exp(-z)]^{-1}$, where $z = x - Vt$.

The relative importance of the effects discussed above depends on the parameters of the system, but under typical conditions (with a low loss and a slight nonlinearity) the first effect—the modulational instability—develops most rapidly. An important exceptional case is that of resonant bubbles. Furthermore, effects such as self-induced transmission may also occur in an inhomogeneous medium with bubbles if the bubbles group in clusters as a result of an instability; this possibility is confirmed by experiment.[7]

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