Nonlinear oscillatory motion of a domain wall in a weak ferromagnet

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Motion of a domain wall (DW) in weak ferromagnets induced by a high-frequency field that is not small is investigated. The nonlinear regime of DW oscillations, in which the maximum velocity of the wall can reach the limiting velocity, is realized experimentally. The proposed theory describes this motion and determines the parameters of the nonlinear DW motion from the data obtained experimentally.

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Nonlinear dynamics of DW in different magnetic materials has recently been the focus of considerable attention. Greatest success has been achieved for weak ferromagnets (WFM) such as rare-earth orthoferrites. Experimental[1,2] and theoretical[3,4] studies, which agree quantitatively, explain the new characteristics of uniform motion of DW in the nonlinear region, i.e., for DW velocities reaching the limiting value.

In this paper we study the nonlinear dynamics of DW under the action of an external periodic magnetic field oriented along the easy axis of magnetization of WFM.

1. We shall describe the DW oscillations theoretically starting from the effective equation describing the dynamics of the magnetization of the WFM.[4,5] This equation can be written in the form

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$$\alpha \Delta \theta - \frac{\alpha}{c^2}\frac{\partial^2 \theta}{\partial t^2} \qquad \beta \sin\theta \cos\theta = \frac{2dh}{\delta} \sin\theta + \frac{\lambda}{2g M_0}\frac{\partial \theta}{\partial t} + \frac{4}{g\delta M_0}\frac{\partial h}{\partial t} . \qquad (1)$$

Here θ is the angular variable for the antiferromagnetism vector, $h = H(t)/M_0$. The remaining notations are the same as in Ref. 4: M_0 is the magnetization of the sublattice; α and δ are the nonhomogeneous and homogeneous exchange constants, respectively; β is the effective anisotropy constant; d is the Dzyaloshinskiĭ constant; and λ is the Gilbert damping parameter. The quantity $c = gM_0\sqrt{\alpha\delta}/2$, which represents the limiting velocity of DW in WFM, coincides with the phase velocity of spin waves in the linear section of the spectrum.

The right side of Eq. (1) is small if the field $h(t)$ is small and the constants λ are small compared to the characteristic parameters of WFM (β, d, δ). We seek a solution of (1) based on perturbation theory for solitons.[6] We assume that $\theta = \theta_0(x,t) + \theta_1 + \dots$, where θ_0 describes a flat DW moving with velocity $V(t)$,

$$\mathrm{tg}\frac{\theta_0}{2} = \exp\left\{ [x - \int^t d\tau\, V(\tau)] \left[1 - \frac{V^2(t)}{c^2}\right]^{-1/2} \right\} . \qquad (2)$$

The condition for the absence of secular terms in the equation for $\theta_1(x,t)$ determines the DW velocity:

$$\frac{dV}{dt} = \left(1 - \frac{V^2}{c^2}\right)^{3/2} \left\{ - \frac{g\lambda\delta M_0 V}{8(1 - V^2/c^2)^{1/2}} + \frac{gdcM_0 h(t)}{\sqrt{\beta\delta}} - \frac{\pi c}{2\sqrt{\beta\delta}}\frac{dh(t)}{dt} \right\} . \qquad (3)$$

The solution of Eq. (3) determines the law of motion of DW under the action of the field. For $H(t) = H\cos\omega t$ we obtain

$$V(t) = \frac{\mu H \cos(\omega t + \gamma)}{[1 + (\mu H/c)^2 \cos^2(\omega t + \gamma)]^{1/2}} . \qquad (4)$$

Here γ is the phase shift, and $\gamma \to 0$ for $\omega \to 0$. The quantity $\mu = \mu(\omega)$ is the mobility of the DW with oscillatory motion and is defined by the equation

$$\mu = \frac{cgd}{\sqrt{\beta\delta}} \left[\frac{1 + \left(\frac{\pi\omega}{gdM_0}\right)^2}{\omega^2 + \omega_r^2} \right]^{1/2} \cong \frac{\mu(0)}{(1 + \omega^2/\omega_r^2)^{1/2}} \qquad \omega_r = \frac{1}{8}\lambda g\,\delta M_0 . \qquad (5)$$

In writing (5) we assumed that $\omega \ll gdM_0 \sim \omega_0$, and ω_0 is the frequency of the antiferromagnetic resonance of WFM. The quantity μ_0 coincides with the usual theoretical value of the mobility of DW in WFM with uniform motion[3,4]: $\mu(0) = (4d/\lambda\delta)\sqrt{\alpha/\beta}$.

It follows from Eq. (4) that the motion of DW is described by a harmonic function only in weak fields ($\mu H \ll c$). As the field increases, the amplitude of the DW velocity V_m is determined by the equation

$$V_m = c(\mu H)/\sqrt{c^2 + (\mu H)^2} \qquad (6)$$

and for $\mu H \gg c$ approaches the limiting value of the DW velocity. At the same time,

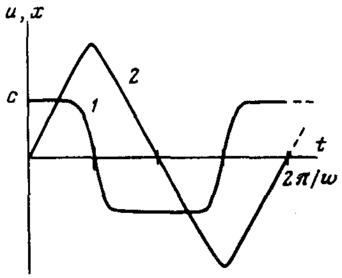

FIG. 1. The function $V(t)$ (curve 1) and $X(t)$ (curve 2) in the nonlinear regime of DW oscillations.

the time dependences of the velocity $V(t)$ (4) and displacement $x(t)$ of DW greatly differ from the harmonic dependences (see Fig. 1).

2. The nonlinear dynamics of DW was studied experimentally in yttrium orthoferrite $YFeO_3$ and iron borate $FeBO_3$ at frequencies up to 150 MHz. The experimental technique and the specimens are similar to those described in Ref. 7.

In the experiment, we measured the quantity $\langle |X(t)| \rangle$, the average value of the absolute magnitude of the displacement of the DW. Quantity $A_0 = (\pi/2)\langle |X(t)| \rangle$ in the harmonic regime is equal to the amplitude of oscillations of DW and determines in a simple way the amplitude of the DW velocity V_m: $V_m = \omega A_0$. However, it can be shown that the quantity A_0 permits determining V_m in the considerably nonlinear regime as well. Using (4), it is easy to obtain[1)].

$$\omega A_0 = \frac{c}{2}\int_0^{(V_m/c)} \frac{dz}{z} \ln\left|\frac{1+z}{1-z}\right| \cong \begin{cases} V_m[1+(V_m/3c)^2], & V_m \ll c \\ \dfrac{c\pi^2}{8} + \dfrac{(c-V_m)}{2}\ln\dfrac{(c-V_m)}{2c}, & V_m \to c \end{cases} . \tag{7}$$

Equation (7) permits determining the true value of V_m from the measured quantity ωA_0 and to construct the dependence $V_m(H,\omega)$. It should be noted that for $\omega A_0 \lesssim c/$

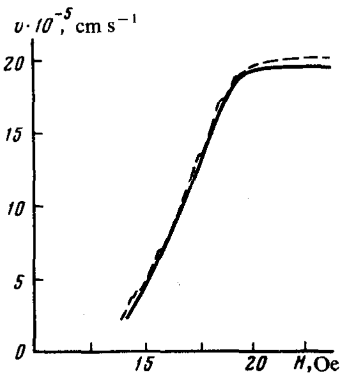

FIG. 2. Experimental dependence $V_m(H)$ for $YFeO_3$ at field frequency 150 MHz. The dashed line represents the dependence $\omega A_0(H)$.

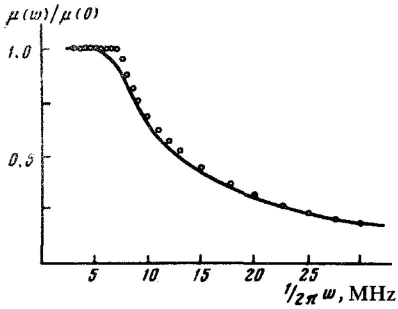

FIG. 3. Dependence of the DW mobility on the frequency of the field. The points show the experimental values and the continuous line is drawn in accordance with Eq. (5). The values of $\mu(\omega)$ are given in relative units and the value of $\mu(0)$ is equal to 3.3×10^3 cm/s Oe (at room temperature).

2 it may be assumed that $V_m = \omega A_0$. Even in the maximally nonlinear regime ($V_m \to c$) the quantity ωA_0 differs from V_m by not more than $(\pi^2/8 - 1)V_m \cong 0.23\ V_m$.

The experimental dependence $V_m(H)$ is presented in Fig. 2. This dependence agrees well with Eq. (6). The value of the limiting DW velocity coincides with the value measured previously using another method.[1,2]

It is interesting to study the interaction of the moving DW with different quasiparticles in the crystal (acoustical or optical phonons, optical or surface magnons). According to theoretical representations, the interaction of DW with quasiparticles is manifested as a characteristic ("shelf") of the dependence $V_m(H)$ for the DW velocity equal to the phase velocity of the quasiparticle.[8,9] It is easy to see that the experimental curve $V_m(H)$ contains a number of characteristics of this type. The characteristics at 4.2 and 7 km/s are determined by interaction with transverse and longitudinal sound.[1,2,8] The technique used, however, reliably reveals an entire series of characteristics of this type (at $V_m = 3.6$ km/s; $V_m = 13.3$ km/s; and, $V_m = 16.3$ km/s). This permits using the proposed technique to search for collective excitations of the system and to analyze their interactions with DW.

The mobility of DW was studied experimentally as a function of frequency. The results are presented in Fig. 3 and agree quantitatively with Eq. (5). Using the relation $\mu(\omega_r) = \mu(0)/\sqrt{2}$, it is possible to find ω_r. A measurement of the quantity $\omega_r = gM_0\lambda\delta/8$ permits determining the relaxation constant λ independent of the standard determination, from the value of the mobility $\mu(0)$.

Thus the high-frequency nonlinear technique is promising for analysis of the basic problems arising when studying nonlinear DW dynamics in magnetic materials.

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[3] It should be noted that although the relationship between $V(t)$ and $X(t)$ (7) is obtained for a specific solution in WFM, it is also valid for all magnetic materials in which the nonlinear state is attributed to the presence of saturation of the velocity and, therefore, the dependences $V(t)$ and $X(t)$ correspond to the graphs in Fig. 1.

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