Low-frequency oscillations of vortices in rotating He II

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The low-frequency branch of the spectrum of oscillations of a stack of disks in rotating He II with large angular velocities corresponds to excitation of an inertial wave, a well known phenomenon in hydrodynamics of rotating classical fluids, in the volume between the disks. This agrees with the results of experiments by Andereck, Chalupa, and Glaberson.

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Andereck, Chalupa, and Glaberson[1,2] have recently performed a series of experiments on torsional oscillations of a stack of disks, in which resonances were observed at frequencies lying much lower than the values that are usually predicted theoretically for such experiments. In this connection, Andereck *et al.* concluded that Tkachenko waves (transverse sound in a lattice of vortices) are excited in their experiments, whose frequency, as is well known, is very small. Without stopping here to analyze in detail this interpretation of the experiments, we indicate only the main problem that has not yet been solved (by the admission of Andereck and Glaberson themselves, see p. 288 in Ref. 2): How can the oscillation of disks, introducing perturbations with wavelengths of the order of the radius of the disks, generate Tkachenko waves whose wavelengths, according to calculation,[1,2] must be an order of magnitude smaller than the radius of the disks? In this paper, we propose another interpretation of the experiments in Refs. 1 and 2, based on the old theory of Hall,[3] which ignores the effect of transverse rigidity of the lattice of vortices, leading to the existence of Tkachenko waves. However, using this theory, it is necessary to reject some approximations that are traditional for the theory, which turn out to be incorrect for the low-frequency branch of the spectrum studied in Refs. 1 and 2. More rigorous calculations performed by us showed that this low-frequency branch lies much lower than previously through and has another asymptotic behavior. In addition, the computed curve of the dependence of the frequency of oscillations on the angular velocity agrees well with the results of the experiments in Refs. 1 and 2.

Hall's theory[3] examined the motion of He II in the space between parallel disks which are, coaxial with the axis of rotation and which execute axial oscillations. Because of the pinning of vortices, the superfluid component is entrained by the disks and a measure of this entrainment is the effective density ρ of the superfluid component (which was introduced by Hall) entrained into oscillations by the disks. This density is defined by the relation [see Eq. (15) in Ref. 3]:

$$\frac{\rho'}{\rho_s} = \frac{(Z_+ + Z_- - 2Z_+Z_-)\alpha^2 + (Z_+ - Z_-)\alpha}{(Z_+ + Z_-)\alpha^2 - (Z_+ - Z_-)\alpha - 2}, \tag{1}$$

where $\alpha = 2\Omega/\omega$, ω is the frequency of the oscillations, Ω is the angular velocity of rotation, $Z_\pm = \tan(k_\pm L)/k_\pm L$, $2L$ is the distance between disks,

$k_{\pm} = \sqrt{(-2\Omega \mp \omega)/\nu_s}$ is the wave number of the flexural wavės propagating along lines of vortices, and ν_s is the flexural rigidity of the vortices. In deriving (1) it was assumed that the ends of the vortices cannot slip along the surface of the disks (total pinning). Hall further simplified Eq. (1), going to the limit $\alpha \to 0$ and $\alpha \to \infty$ [Eqs. (16) and (17) in Ref. 3]. Below we used these simplified equations in interpreting experiental data although they were criticized in Ref. 4. In particular, it follows from them that the physical resonances (poles of the function ρ'/ρ_s) exist only in the region $\omega > 2\Omega$ and are determined by the poles of the function $\tan(k_- L)$, i.e., by the condition that a half-integral number of wave fits into the length of a vortex with wavelength k_-. This leads to linear dependence of the frequency ω on the angular velocity Ω:

$$\omega = \nu_s\left(\frac{\pi}{2}\,\frac{2n-1}{L}\right)^2 + 2\Omega , \tag{2}$$

where $n = 1,2,...$ is the number of the branch in the spectrum. However, a more rigorous calculation using the starting expression (1) shows that the shape of the lower branch of the spectrum $n = 1$ is very different. Figure 1 shows for the two lower branches of the spectrum $n = 1$ and $n = 2$ the old dispersion curves, determined by expression (2), and the new curves obtained by a numerical calculation of the poles of

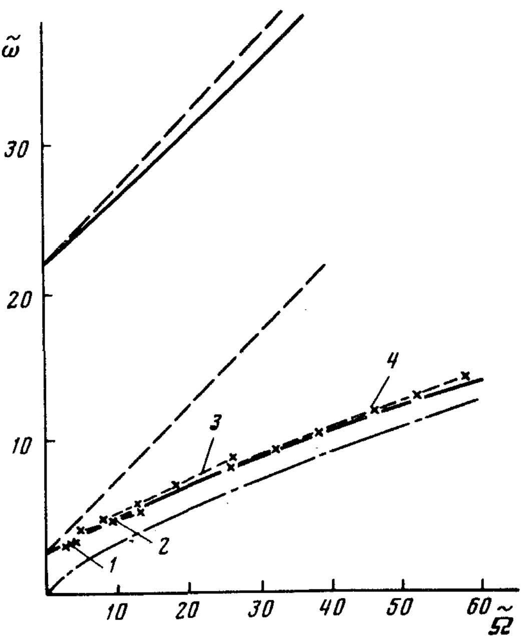

FIG. 1. Low-frequency branches of the spectrum $n = 1$ and $n = 2$. The continuous lines correspond to the poles of the function ρ'/ρ_s, determined by Eq. (1); the dashed lines correspond to the poles of the function $\tan(k_- L)$[Eq. (2)]; the dot-dashed line corresponds to the asymptotic curve [Eq. (3)] which the $n = 1$ branch approaches as $\tilde{\Omega} \to \infty$. The dashed lines with the cross marks are drawn through the experimental points obtained for different distances d between disks: $1 - d = 0.0208$ cm, $2 - d = 0{,}0366$ cm, $3 - d = 0.0508$ cm, $4 - d = 0.0762$ cm; for definition of the dimensionless parameters $\tilde{\omega}$ and $\tilde{\Omega}$ for these points it was assumed that $v_s = 10^{-3}$ cm^2 s^{-1}.

the function ρ'/ρ_s given by expression (1). The graph is constructed in dimensionless variables $\widetilde{\omega} = \omega L^2/\nu_s$ and $\widetilde{\Omega} = \Omega L^2/\nu_s$. It is evident that the new curve for $n = 1$ lies much lower than the old curve and it has a different asymptotic behavior for $\widetilde{\Omega} \to \infty$, determined by the expression

$$\widetilde{\omega} = (2\widetilde{\Omega})^{3/4} . \quad (3)$$

In dimensional variables, the value of this expression coincides with the frequency $\omega = 2\Omega/\sqrt{AL}$, obtained in Ref. 5, of characteristic oscillations of He II in a cylindrical vessel for the case in which the Tkachenko rigidity of the lattice of vortices can be ignored [see Eq. (43) in Ref. 5, and in addition, $A = \sqrt{2\Omega/\nu_s}$ for the case of total pinning being examined here]. According to Ref. 5, this mode of oscillations correspond to excitation of waves in the bulk with spectrum $\omega^2 = (2\Omega)^2 p^2/(p^2 + q^2)$ (p and q are the projections of the wave vector on the axis of rotation and on the plane perpendicular to it). This is a well-known mode in the case of a classical rotating fluid and is called an inertial wave. Thus the mode corresponding to the lower branch of the spectrum in our problem goes over into a purely inertial wave as Ω increases. It is for this reason that the transition from (1) to the simplified Hall equations is not accessible for this branch, since it can be shown that with such a transition an inertial wave, which was actually incorporated by Hall in his derivation of (1), although he did not explicitly write it out and did not discuss it, is discarded in the bulk. The inertial wave in Hall's theory corresponds to a velocity field constant in space, since he examined the limiting case of disks with a large radius, when $q \to 0$ and this means $p \to 0$ as well, if the quantity ω/Ω is finite.

Figure 1 shows the frequencies of the resonances observed in the experiments by Andereck *et al.*[1,2] for different distances $d = 2L$ between disks. These frequencies are situated quite close to the theoretical dispersion curve for the lower branch of the spectrum. The experimental data for $d = 0.269$ cm, not shown on the graph, since they correspond to very large $\widetilde{\Omega} \gtrsim 170$, also showed no worse agreement with theory. The agreement leads to the conclusion that the vibrational mode related to excitation of an intertial wave in the bulk was observed in the experiments performed by Andereck *et al.*. This same mode was observed previously in the experiments in Ref. 6, but in another region of parameters, in which the slipping of vortices cannot be ignored. Experiment and theory show that this mode must gradually transform from the intertial wave into a Tkachenko wave, although this occurs for very large values of the ratio L/R^2 (R is the radius of the disks or vessel).[5,7]

[1]C. D. Andereck, L. Chalupa, and W. I. Glaberson, Phys. Rev. Lett. **44**, 33 (1980); W. I. Glaberson, Physica B + C **109**, 1567 (1982).

[2]C. D. Andereck and W. I. Glaberson, J. Low Temp. Phys. **48**, 297 (1982).

[3]H. E. Hall, Proc. Roy. Soc. **245**, 546 (1958).

[4]E. L. Andronikashvili, Yu. G. Mamaladze, S. G. Matinyan, and D. S. Tsakadze, Usp. Fiz. Nauk **73**, 2 (1961) [Sov. Phys. Usp. **4**, 1 (1961/62)].

[5]É. B. Sonin, Zh. Eksp. Teor. Fiz. **70**, 1970 (1976) [Sov. Phys. JETP **43**, 1027 (1976)].

[6]D. S. Tsakadze and S. D. Tsakadze, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 605 (1973) [JETP Lett. **18**, 355 (1973)]; S. D. Tsakadze, Zh. Eksp. Teor. Fiz. **71**, 754, (1976) [Sov. Phys. JETP **44**, 398 (1976)].

[7]S. D. Tsakadze, Fiz. Nizk. Temp. **4**, 148 (1978); J. S. Tsakadze, S. J. Tsakadze, and É. B. Sonin, Phys. Rev. B **21**, 3028 (1980).

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