Orbital angular momentum in B phase of ^{3}He and its effect on the texture in a rotating vessel

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The B phase of ^{3}He in a magnetic field **H** has, in addition to the spin angular momentum $\mathbf{S}\sim\mathbf{H}$, an orbital angular momentum $L_i\sim R_{ik}S_k$, where R_{ik} is the rotational matrix entering into the order parameter of ^{3}He-B $\Delta(T)R_{ik}e^{i\phi}$. A change in the energy of the fluid $\vec{\Omega}\,\mathbf{L}\sim\Omega_i R_{ik}H_k$ caused by the rotation of the vessel with angular velocity $\vec{\Omega}$ has an orientating effect on the order parameter, comparable in magnitude to the orienting effect of vortices. It is proposed that the magnitude of **L** be measured from the shift of the NMR spectrum in rotating ^{3}He-B.

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The orbital angular momentum in superfluid ^{3}He has always been assumed to be an attribute of the anisotropic phase A, in which all Cooper pairs have the same momentum, oriented along the anisotropy axis (see, for example, Ref. 1). However, an experiment in which the orbital angular momentum of ^{3}He-A is measured has not yet been performed. We shall show that the isotropic phase B, in the presence of a magnetic field, also has an orbital angular momentum and, in addition, it is much easier to observe it experimentally than in the A phase.

The B phase has a unique breakdown of symmetry relative to rotations in spin and coordinate spaces. In particular, the state of the system characterized by the order parameter $\Delta(T)R_{ik}e^{i\phi}$ varies with rotation of both the spin and coordinate spaces, but does not vary with simultaneous matched rotation of both spaces. The generator of rotations, which do not change the order parameter, has the form

$$\hat{J}_i = \hat{L}_i + R_{in}\hat{S}_n \tag{1}$$

where $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$ are generators of rotations in the orbital and spin space, respectively. The action of the operator $\hat{\mathbf{J}}$ on R_{ik} gives

$$\hat{J}_i R_{kl} = \hat{L}_i R_{kl} + R_{in}\hat{S}_n R_{kl} = i(e_{ikm}R_{ml} + R_{in}\,e_{nlm}R_{km}) = 0. \tag{2}$$

The operator $\hat{\mathbf{J}}$ corresponds to the total angular momentum operator of a Cooper pair. If the coordinate systems are chosen in the spin and coordinate spaces, rotated relative to each other with the help of the matrix R_{ik}, then in this case $\hat{\mathbf{J}}=\hat{\mathbf{L}}+\hat{\mathbf{S}}$. Equality (2) means that the Cooper pairs in the B phase are located in the quantum state with $J=0$, as a result of which the fluid is isotropic.

Because of the rigidity of the quantum state of Cooper pairs, we can expect that if a pair spin density $\mathbf{S}^p$ appears in the B phase, for example, in an external magnetic field, then there simultaneously arises an orbital angular momentum density

$$L_i = -R_{ik} S^p_k \,, \qquad (3)$$

so that the total angular momentum **J** remains equal to zero and the fluid remains isotropic (see also Refs. 2 and 3).

Relation (3) can be checked by calculating the superfluid current density in the B phase. We first examine $T=0$. In this cases the current density j must, aside from a gradient term $\rho \mathbf{v}_s = \hbar \rho \vec{\nabla} \phi / 2m$, contain also terms related to the angular momentum **L**:

$$\mathbf{j} = \rho \mathbf{v}_s + \frac{1}{2} \operatorname{rot} \mathbf{L} \,. \qquad (4)$$

Using the gradient expansion of the Gor'kov equations (see Ref. 4), we find the current in the B phase in an inhomogeneous magnetic field under the condition that $R_{ik} = \text{const}$. In this case, we indeed obtain (4) with

$$L_i = -\frac{\chi_B(T=0)}{\gamma} R_{ik} H_k \,, \qquad (5)$$

where $\chi_B(T=0)$ is the susceptibility of the B phase at $T=0$, and γ is the gyromagnetic ratio. It is easy to see that expressions (5) and (3) are equivalent, since at $T=0$ there are no excitations and the spin of the fluid $\mathbf{S} = \frac{\chi_B}{\gamma} \mathbf{H}$ coincides with the spin of a pair $\mathbf{S}^p$.

Let us now consider $T \neq 0$. Here the result depends on which momentum **L**, dynamic or static, is of interest. If we are interested in the dynamic momentum, then in calculating the current it is not necessary to include the reaction of excitations to the gradient field, since the excitations do not have time to relax to the new equilibrium distribution. In this case, we obtain for **L** the following expression:

$$L_i = -\frac{2}{3} \frac{\chi_n}{\gamma} (1 - f(T)) R_{ik} H_k \,, \qquad (6)$$

$$f(T) = \int_0^\infty d\epsilon \, \frac{\epsilon^2}{E^2} \, \frac{1}{2T} \operatorname{ch}^{-2} \frac{E}{2T}$$

($E = \sqrt{\varepsilon^2 + \Delta^2}$ is the spectrum of excitations, χ_n is the susceptibility of the normal Fermi liquid), which coincides with (3), if the expression for $\mathbf{S}^p$, found by Leggett and Takagi,[5] is substituted into it. In the static case, which we are interested in, it is necessary to include the contribution of excitations to the momentum **L**. Then **L** is given by expression (6), in which $f(T)$ must be replaced by the Yosida's function $Y(T) = \int_0^\infty d\varepsilon (1/2T) \operatorname{ch}^{-2} E/2T$. Taking into account the Fermi fluid also changes the coefficient in front of $R_{ik} H_k$ in (6).

The angular momentum **L** can be found from its effect on the orientation of the order parameter in a rotating vessel, where due to **L** there arises an additional term in the energy

$$F_L = -\vec{\Omega} \mathbf{L} \sim \frac{\chi^p}{\gamma} \Omega_i R_{ik} H_k \,. \qquad (7)$$

Here $\vec{\Omega}$ is the angular velocity of rotation of the vessel. This term describes the orienting action of $\vec{\Omega}$ and **H** on the order parameter matrix

$$R_{ik}(\mathbf{n},\theta_0) = \frac{1}{4}(-\delta_{ik} + 5 n_i n_k + \sqrt{15}\, e_{ikl} n_l) \tag{8}$$

determined by the axis of rotation **n** and the angle of rotation $\theta_0 = \arccos(-1/4)$, rigidly fixed due to the spin-orbital interaction.

Although for attainable angular velocities $\Omega \sim 1$ rad/s the energy (7) is small, it competes with other orienting actions: energy of magnetic anisotropy in ^{3}He-B $F_M = -a(\mathbf{n},\mathbf{H})^2$, arising due to the small spin-orbital interaction, so that $a \sim 10^{-6}\chi$ (see, for example, Ref. 6), and the energy of interaction with vortices $F_v = (2/5)a\lambda [(\Omega_i/\Omega)R_{ik}H_k]^2$, due to the breakdown in anisotropy inside the nucleus of a vortex[7] ($\lambda \lesssim 1$ at $\Omega \sim 1$ rad/s). It is evident that for characteristic fields $H \sim 300$ G, used in experiments[8] ($\gamma H \sim 1$ MHz), the orientational energy (7) is comparable to F_M and F_v. The energy F_L becomes dominant in low fields. However, even in high fields the presence of **L** can be observed from the change in the NMR spectrum with inversion of the direction of rotation or field, since in the absence of **L** the spectrum does not change.

The orbital angular momentum **L** can be found by measuring the shift in the NMR frequency in rotating ^{3}He-B,[8] proportional to $\sin^2\beta$, where β is the angle between **H** and **n**. According to Ref. 9, where the magnetic fields oriented at an angle to the rotational axis is examined, we shall find the equilibrium angle β by minimizing the orientational energy $F_L + F_M + F_v$, which is conveniently done by writing F_L from (7) in terms of the parameter λ

$$F_L = \frac{4}{5} a \tilde{\lambda} \frac{\Omega_i}{\Omega} R_{ik} H_k . \tag{9}$$

We have

$$\lambda \left(u \cos 2\mu \pm \frac{u^2 - 1/2}{\sqrt{1-u^2}} \sin 2\mu\right) + \frac{\tilde{\lambda}}{H}\left(\cos\mu \pm u \frac{\sin\mu}{\sqrt{1-u^2}}\right) = 1, \quad u = 1 - \frac{5}{4}\sin^2\beta, \tag{10}$$

which becomes Eq. (2) in Ref. 9 at $\tilde{\lambda} = 0$. Here μ is the angle between **H** and $\vec{\Omega}$. Of the two solutions (10), which correspond to different signs $\pm$, we must choose the solution which gives the minimum energy. Measuring $\sin^2\beta$ from the shift of the NMR frequency with different μ, it is possible to find λ and $\tilde{\lambda}$ simultaneously, i.e., to determine both the effect of vortices on the orientation of the order parameter and the magnitude of the orbital angular momentum **L**. It is especially important to measure the shift in the absorption NMR frequency in a transverse field ($\mu = \pi/2$), where it occurs only due to the orbital angular momentum. In this case, at intermediate values of $\tilde{\lambda}/H$, Eq. (10) gives

$$\sin^2\beta \approx \frac{4}{5}\,\frac{\tilde{\lambda}^2}{H^2\,(1+\lambda)^2}\,. \qquad (11)$$

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