Growth of total cross sections and the quark structure of hadrons

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(Submitted 20 November 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 2, 113–115 (20 January 1983)

The dependence of the total cross sections on the masses of the valence quarks is discussed.

PACS numbers: 12.35.Ht, 13.85.Lg

A simple method for incorporating the composite structure of hadrons in constructing the kernel $U(s, t)$ of the three-dimensional integral dynamic equation[2] $F = F[U]$ for the elastic scattering amplitude of hadrons h_1 and h_2 was discussed in Ref. 1. That method works from the assumption of a quasi-independent scattering of the valence quarks in an effective field which arises in the course of the interaction of the hadrons. The equation for the amplitude $F(s, t)$ in the impact-parameter representation leads to the expression

$$F(s, t) = \frac{s}{\pi^2} \int_0^\infty b\,db \, \frac{u(s, b)}{1 - iu(s, b)} J_0\left(b\sqrt{-t}\right), \tag{1}$$

where $u(s, b)$ is the Fourier-Bessel transform of the function $U(s, t)$. For the function $u(s, b)$ we accordingly use

$$u(s, b) = \prod_{i=1}^{n_1} f_i(s_i, b) \prod_{j=1}^{n_2} f_j(s_j, b), \tag{2}$$

where $f_i(s_i, b)$ is the scattering amplitude for the ith valence quark in the effective field, and n_1 and n_2 are the numbers of valence quarks in hadrons h_1 and h_2, respectively. The following expression was chosen for the quark-scattering amplitude $f_i(s_i, b)$:

$$f_i(s_i, b) = g_i(s_i) e^{-\mu b}. \tag{3}$$

This expression yields an amplitude $F(s, t)$ having the correct analytic structure in the $\cos\theta$ plane. The energy dependence is adopted in the form $g_i(s_i) = g_i s_i^{\lambda}$; the polynomial boundedness of the generalized reaction matrix $U(s, t)$ and the requirement of an asymptotic growth of the total cross sections are taken into account. The parameter μ, which determines the quark-interaction radius, is assumed independent of the quark species.[1] There is, however, another possibility: to introduce different interaction radii for the quarks of different masses. This assumption turns out to be a natural one in a study of diffractive dissociation and the production of new heavy hadrons.[3] The parameter μ_i in this case is assumed equal to the mass of the corresponding quark: $\mu_i = m_i$.

Also using (2) for $u(s, b)$ we find

$$u(s, b) = i C s^{\lambda N} \exp(-M b), \tag{4}$$

where

$$N \equiv N_{h_1 h_2} = \sum_i n_i = n_1 + n_2, \quad M \equiv M_{h_1 h_2} = \sum_i m_i n_i,$$

and

$$C \equiv C_{h_1 h_2} = \frac{1}{M^2}{}^{\lambda N} \prod_{i=1}^{n_1} g_i m_i^{2\lambda} \prod_{j=1}^{n_2} g_j m_j^{2\lambda}. \tag{5}$$

It follows from (4) and (5) that N, M, and C depend on the quark composition of h_1 and h_2 and on the masses of the valence quarks.

The total cross section for the interaction of the hadrons h_1 and h_2 can now be written

$$\sigma_{tot}^{h_1 h_2}(s) = 4\pi R_{h_1 h_2}(s)$$
$$= 4\pi \left(\frac{N}{M}\right)^2 \left[\lambda^2 \ln^2 s + 2\lambda \frac{\ln C}{N} \ln s + \frac{\ln^2 C}{N^2}\right]. \tag{6}$$

The growth rate of the total cross sections and their relative magnitude are thus determined by the quark composition of the interacting hadrons.

From (6) we can extract several experimentally testable predictions. Let us assume that the u and d quarks have the same mass, which we denote by m_g. We then have the asymptotic relation

$$\frac{\sigma_{tot}(K_p)}{\sigma_{tot}(\pi p)} = \left(\frac{5}{m_s/m_q + 4}\right)^2 .$$

The ratio m_s/m_q is an adjustable parameter. Using the experimental data on the ratio $\sigma_{\rm tot}(Kp)/\sigma_{\rm tot}(\pi p)$, we find $m_s/m_q = 1.5$. Adopting this value, we find the following for the ratios of the total cross sections for the interactions of hyperons with protons:

$$\frac{\sigma_{tot}(\Sigma p)}{\sigma_{tot}(pp)} = \left(\frac{6}{m_s/m_q + 5}\right)^2 = 0.85,$$

$$\frac{\sigma_{tot}(\Xi p)}{\sigma_{tot}(pp)} = \left(\frac{6}{2m_s/m_q + 4}\right)^2 = 0.73.$$

These values agree well with experimental data. The inequality $m_s > m_q$ leads to

$$\sigma_{tot}(Kp) < \sigma_{tot}(\pi p), \quad \sigma_{tot}(\Xi p) < \sigma_{tot}(\Sigma p) < \sigma_{tot}(pp) .$$

In the additive-quark model[5] the ratio of cross sections depends on only the number of valence quarks. For the ratio $\sigma_{\rm tot}(\pi p)/\sigma_{\rm tot}(pp)$ for example, this model yields the value 2/3. The present paper predicts an asymptotic value of unity for this ratio. The ratios of cross sections for other reactions differ from unity because of differences in the masses of the valence quarks making up the hadrons.

Knowing the ratio m_Q/m_q we can generate predictions of the total cross sections for the interactions of particles containing heavy quarks. If, for example, we set[4] $m_c/m_q = 10$, we find the value 0.16 for the ratio $\sigma_{\rm tot}(\Lambda_c p)/\sigma_{\rm tot}(pp)$. Consequently, the cross section $\sigma_{\rm tot}(\Lambda_c p)$ at $p_L = 100$–200 GeV/c should be about 6 mb. A corresponding estimate for the cross section $\sigma_{\rm tot}(\psi p)$ yields 1 mb.

In this approach, the total cross sections grow at the maximum possible rate, in proportion to $\ln_2$ s. The coefficient of this double logarithm depends on the number and masses of the valence quarks. For the total cross sections for the πN and NN interactions we have

$$\sigma_{tot}^{(\infty)}(s) = \frac{4\pi\lambda^2}{m_q^2}\ln^2 s , \tag{7}$$

where m_q is the mass of the $u(d)$ quark. If we use the Froissart bound[6] $\sigma_{\rm tot}(s) \leqslant (\pi/m_\pi^2)\ln^2 s$, we find a lower limit on the mass of the $u(d)$ quark:

$$m_q \geqslant 2\lambda m_\pi . \tag{8}$$

The parameter λ is related to the rate at which the scattering cross section falls off at large angles; as was shown in Ref. 1, the choice $\lambda = 1/2$ leads to a good agreement with experimental data on πN and NN scattering. The limitation on the quark mass then takes the simple form $m_q \geqslant m_\pi$.

In summary, we have derived the dependence of the cross sections on parameters related to the hadron structure: the number and masses of the valence quarks. The dependence of the cross sections on the quark masses explains the difference in the magnitudes of the cross sections on the basis of different quark compositions of the hadrons. The limitation on the growth of the total interaction cross sections found from the general principles of field theory suggests that the mass of the $u(d)$ quark must have a lower limit of the π-meson mass. The equations and conclusions of this study are based on certain assumptions. The assumption of a quasi-independent scattering of the valence quarks is a natural one in view of the property of asymptotic freedom. The form of the quark amplitude $f_q(s_q,b)$ is determined by the analytic properties of the generalized reaction matrix, which follow from the known analytic properties of the amplitude in terms of the variable $\cos\theta$. Essentially the basic assumption used here is to identify the quark interaction radius with the reciprocal of the quark mass.

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Translated by Dave Parsons
Edited by S. J. Amoretty