Interaction of a light particle with a system of heavy particles

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(Submitted 25 November 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 3, 129–131 (5 February 1983)

When a light particle interacts with a bound complex, the decay over distance is described by a $1/r^2$ law instead of $1/r^4$ law. The corresponding force is unusual in several respects: It is proportional to the mass of the particle; it acts only in the s state of the particle; it generates a series of bound states similar to Efimov levels, etc.

PACS numbers: 05.60. + w

The well-known expression[1]

$$V(r) = -e^2\alpha(0)/(2r^4) \tag{1}$$

describes the interaction of a van der Waals particle of charge e with a bound complex of polarizability

$$\alpha(\omega) = 2\sum_n{}' |\langle 0|d_z|n\rangle|^2 \epsilon_n/(\epsilon_n^2 - \omega^2 - i\delta)$$

($\mathbf{d}$ is the dipole moment, and ε_n is the excitation energy) over distances greater than the radius of the complex, R. Expression (1) is customarily applied to electron-plus-atom systems, for which the parameter $m\varepsilon R^2$ is on the order of unity (m is the mass of the particle, ε is a scale value of ε_n, and $\hbar = 1$).

It turns out that for a very light particle,

$$m\epsilon R^2 \ll 1 \tag{2}$$

[for the pion-deuteron system and the electron-(muonic atom) system, for example], expression (1) holds only at

$$r \gg (m\epsilon)^{-1/2}, \tag{3a}$$

and changes radically at

$$R \ll r \ll (m\epsilon)^{-1/2}. \tag{3b}$$

1. The Schrödinger equation for the scattering of a light particle by a complex is[1)]

$$[-\Delta_{\mathbf{r}}/2m + e\,\mathbf{d}\nabla(1/r) + H_c - E_c - k^2/2m]\Psi = 0, \tag{4}$$

where $\mathbf{k}$ is the relative momentum, and H_c and E_c are the Hamiltonian and energy of the complex, for which the wave function of the internal motion is ϕ.

The potential V describes the effect of the polarization of the complex on the motion of the particle, averaged over the internal motion of the complex. Corresponding to the motion of the particle is the wave function $\psi(r)$, which is the projection of Ψ onto the state ϕ. The corresponding projection of Eq. (4) gives us

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$$(H_0 - k^2/2m)\psi = 0, \quad H_0 = -\Delta/2m + V(r), \qquad V(r) = e\nabla(1/r)\langle \mathbf{d}\hat{\xi}\rangle, \tag{5}$$

where we have set $\Psi = \hat{\xi}\psi\phi$; hence $\langle\hat{\xi}\rangle = 1$, and the angle brackets denote an average over the state ϕ. The potential V generally depends on the momentum operator $\mathbf{p}$, which acts on the wave function ψ.

2. For a "rigid" complex, for which the condition $me(\langle d^2\rangle)^{1/2} \ll 1$ holds, we have

$$\hat{\xi} = 1 + [E_c + k^2/2m - H_c - H_0]^{-1} e\mathbf{d}\nabla(1/r)$$

and Eqs. (5) yield

$$V(r) = -\frac{e^2}{\pi}\int_0^\infty d\omega \, \mathrm{Im}\,\alpha(\omega)\nabla(1/r)[\omega - (\Delta + 2i\mathbf{p}\nabla)/2m]^{-1}\nabla(1/r). \tag{6}$$

The first term in the denominator in (6) is predominant in region (3a); this circumstance leads us directly to (1) by virtue of the sum rule $\alpha(0) = (2/\pi)\int_0^\infty (d\omega/\omega)\,\mathrm{Im}\,\alpha(\omega)$. In region (3b), on the other hand, where the opposite situation prevails, the $\mathbf{p}$ dependence of (6) is described by a factor

$$O = \frac{\sin(pr)}{pr}\exp(-i\,\mathbf{pr}) = \delta_{l,0},$$

which singles out the projection of the wave function ψ onto the s state of the relative motion of the particle and the complex. Using the sum rule

$$\int_0^\infty d\omega\,\mathrm{Im}\,\alpha(\omega) = \pi\langle \mathbf{d}^2\rangle/3$$

we then find

$$V(r) = -me^2\langle \mathbf{d}^2\rangle O/(3r^2). \tag{7}$$

3. Potential (7) depends on the mass of the particle, and this dependence makes the potential unusual: It is generated by the excitation of the *relative* motion of the particle and the complex by fluctuations of the dipole moment of the complex, which are sensed by the particle specifically because it is light (an adiabatic situation). Furthermore, potential (1) is the result of the excitation by the particle of *internal* motion in the complex whose inverse effect the particle experiences.

The very nature of the mass dependence of the potential is remarkable: The force acting on the particle is proportional to its mass, so that all the particles move in the field of the complex in an identical way under condition (2) (an "equivalence principle" for van der Waals and inertial forces). This property, which has been thought to be peculiar to the gravitational and inertial forces, is also exhibited by simple Coulomb systems.

4. If the complex has a large polarizability ($\lambda = me(\langle d^2\rangle)^{1/2} \gtrsim 1$), then the operator $\hat{\xi}$ in region (3b) is, with respect to the s state, simply $f(x)$, where $x = \mathbf{d}\mathbf{r}/dr$, and

$$\frac{d}{dx}\left[(1-x^2)\frac{df}{dx}\right] + (2medx - \sigma)f = 0, \quad \sigma = 2me\langle dxf\rangle, \quad V(r) = -\sigma/(2mr^2). \tag{8}$$

The $1/r^2$ law thus holds for arbitrary values of λ in region (3b).[2)]

At small values of λ, Eqs. (8) yield $\sigma = 2\lambda^2/3$, returning us to (7). At a certain $\lambda = \lambda^0 \sim 1$, which depends on the structure of the complex, we reach the critical value $\sigma = 1/4$, which corresponds to "central incidence"[1] (in this case, "incidence on a complex"). At large values of λ we have

$$\psi \propto \cos[(\sigma - 1/4)^{1/2} \ln(r/R) + \text{const}]$$

and a series of bound states arises, in a number equal to the number of zeros of ψ in region (3b):

$$\frac{(\sigma - 1/4)^{1/2}}{2\pi} \ln(1/m\epsilon R^2).$$

The energies E_n obey the similarity law $E_n/E_{n-1} = \text{const}$.

5. The situation is reminiscent of that in a system of three resonantly interacting particles, for which, in the region $r_0 \ll \mathscr{R} \ll a$ ($\mathscr{R}$ is the radius of the system, a is the scattering length, and r_0 is the range of the forces), a potential $1/\mathscr{R}^2$ also arises, as does a series of bound states—Efimov levels—with properties similar to those that we have just described.[2]

We might thus expect some analog of Efimov levels in atomic physics also, e.g., in weakly bound molecular systems. The corresponding conditions are

$$\epsilon \ll e\bar{e}/R, \qquad (e\bar{e}R)^{-1} \lesssim m \ll (\epsilon R^2)^{-1}, \tag{9}$$

where $\bar{e}$ is the effective charge of the complex ($\langle d^2 \rangle = \bar{e}^2 R^2$).

We wish to thank the participants of the theoretical seminar at the Lebedev Physics Institute for discussions.

[1)] We are ignoring the higher-order multipoles, which fall off rapidly with r. For simplicity, we are assigning the highest symmetry to the complex (the expectation values of the angular and multipole moments are zero). We are also omitting the purely Coulomb term in the interaction of the particle with the complex.
[2)] This point will be discussed in more detail in a separate paper to be published in the near future.

[1] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974 (Quantum Mechanics: Non-Relativistic Theory, Pergamon, New York, 1977).
[2] V. N. Efimov, Yad. Fiz. **12**, 1080 (1970) [Sov. J. Nucl. Phys. **12**, 589 (1971)].

Translated by Dave Parsons
Edited by S. J. Amoretty