Parametric interaction of oppositely propagating light waves of identical frequency

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Waves of identical frequency in media exhibiting a reactive (nonabsorbing) nonlinearity may exchange energy with each other. The exchange results from a spatial modulation of the interfering light fields by the gyration vector of the magnetic medium. There is a threshold in the dependence of the gain on the wave power; the gain also depends on the deviation of the wave polarization from circular. The gain reaches a maximum in the case of a linearly polarized intense wave and vanishes in the case of a circular polarization.

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Working from conservation laws, Zel'dovich[1] showed that energy could not be exchanged between light waves of identical frequency propagating in opposite directions. Such an energy exchange in a reactive isotropic medium would constitute a violation of momentum conservation.

In this letter we predict a parametric interaction of light waves having identical frequencies and propagating in opposite directions by a mechanism which stems from the presence of a static magnetic field **H** in the interaction medium. This field may either by applied externally or be the consequence of the spontaneous magnetization of the medium, **M**.

The dielectric function of such a medium is[2]

$$\epsilon_{ik} = \epsilon_0 \delta_{ik} + i e_{ikl} g_l , \tag{1}$$

where e_{ikl} is the Levi-Civitas density, and **g** is the gyration vector, which is related to the magnetic field by $\mathbf{g} = f\mathbf{H}$, where f is a constant determined by the properties of the medium.

We assume that light waves are propagating in opposite directions along the z axis. We write the complex electric field **E** of these waves, which is related to the real field $\mathbf{E}_{\text{real}}$ by $\mathbf{E}_{\text{real}} = 0.5\,[\mathbf{E}(z,t) + \mathbf{E}^*(z,t)]$, in the form

$$\mathbf{E} = e^{-i\omega t - ikz}\,[\,\mathbf{e}_+ E_{L+}(z)\, e^{i\Phi z} + \mathbf{e}_- E_{L-}(z)\, e^{-i\Phi z}]$$

$$+ e^{-i\omega t + ikz}\,[\mathbf{e}_+ E_{s+}(z) e^{-i\Phi z} + \mathbf{e}_- E_{s-}(z)\, e^{i\Phi z}]\ . \tag{2}$$

In (2), $e_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$ are circular-polarization unit vectors, and $\Phi = (\omega/2c\sqrt{\varepsilon_0})(\,\mathbf{g}\mathbf{e}_z)$. For definiteness, we assume $k > 0$, i.e, that the signal wave E_s is propagating in the positive direction along the z axis. The slight dependence of the amplitudes E_L and E_s on z axis results from the nonlinear interaction.

In the simplest case, the change in dielectric function (1) caused by the effect of light field (2) on the magnetic properties of the medium is

$$\delta\epsilon_{ik} = i e_{ikl}\delta g_l \equiv i e_{ikl}\, \eta_l \left\{ E^*_{L+} E_{s+} e^{2i(k-\Phi)z} + E^*_{L-} E_{s-} e^{2i(k+\Phi)z} \right\} + \ldots . \tag{3}$$

In (3) we have explicitly write out only those terms describing the perturbations of the dielectric function in which we are interested here, whose scattering of the pump waves E_L gives rise to the nonlinear increment in the electric displacement of wave E_s: $\delta D_{si} = \delta\epsilon_{ik} E_{Lk}$. Writing $\delta\mathbf{D}_s$ as $\delta\mathbf{D}_s = \mathbf{e}_+ \delta D_+ + \mathbf{e}_- \delta D_-$, we find from (3)

$$\begin{aligned} \delta D_+ &= -(\vec{\eta}\mathbf{g}) \left\{ |E_{L+}|^2 E_{s+} + E_{L+} E^*_{L-} E_{s-} e^{4i\Phi z} \right\}, \\ \delta D_- &= (\vec{\eta}\mathbf{g}) \left\{ |E_{L-}|^2 E_{s-} + E^*_{L+} E_{L-} E_{s+} e^{-4i\Phi z} \right\}. \end{aligned} \tag{4}$$

The truncated equations for media with a reactive nonlinearity can be put in the following form, where we are using (4):

$$\begin{aligned} \frac{dE_{s+}}{dz} &= i\rho \left\{ |E_{L+}|^2 E_{s+} + E^*_{L-} E_{L+} E_{s-} e^{4i\Phi z} \right\}, \\ \frac{dE_{s-}}{dz} &= -i\rho \left\{ |E_{L-}|^2 E_{s-} + E_{L-} E^*_{L+} E_{s+} e^{-4i\Phi z} \right\}, \end{aligned} \tag{5}$$

where $\rho = (\omega^2/2c^2k)(\vec{\eta}\mathbf{g})$.

Assuming $|E_L|^2 \approx \text{const}$, we seek a solution of Eq. (5) in the form

$$E_{s+} = A\, e^{(l+2i\Phi)z}, \quad E_{s-} = B\, e^{(l-2i\Phi)z}. \tag{6}$$

Substitution of (6) into (5) leads to an algebraic system of homogeneous equations for A and B; the condition for the existence of a nontrivial solution of this system determines the value of l:

$$l_{1,2} = \frac{i}{2}\rho\zeta\, |\mathbf{E}_L|^2 \pm \sqrt{2\Phi\rho\, |\mathbf{E}_L|^2 - 4\Phi^2 - \frac{1}{4}\rho^2\zeta^2 |\mathbf{E}_L|^4}, \tag{7}$$

where $\zeta = (|E_{L+}|^2 - |E_{L-}|^2)/|\mathbf{E}_L|^2$ is the eccentricity of the pump wave E_L. The gain is related to l by $g_s = 2 \max \operatorname{Re}(l_{1,2})$. In the case of a time-varying perturbation of the medium by light fields with different frequencies, $\delta\hat{\epsilon}$ has an imaginary part because of the nonzero response time of the medium. In our case the perturbations $\delta\hat{\epsilon}$ are static, so that ρ is real. The gain could thus be different from zero only by virtue of a positive quantity in the radical. For a linearly polarized pump wave we have $\zeta = 0$, and from (7) we find

$$g_s = 2|\Phi| \sqrt{\frac{P}{P_{cr}} - 1}, \tag{8}$$

where P is the power of the pump wave E_L, and P_{cr} is the critical power, at which g_s

becomes nonzero:

$$P_{cr} = \frac{c\sqrt{\epsilon_0}}{4\pi}\left|\frac{\Phi}{\rho}\right| . \qquad (9)$$

We thus conclude that energy can be transferred only from a wave whose intensity exceeds threshold (9). For a gain to exist, the $\Phi\rho$ must be positive. Since $\text{sign}(\Phi\rho) = \text{sign}\{(\mathbf{e}_z\mathbf{g})(\vec{\eta}g)\}$ and $\vec{\eta} \propto \delta g$, this condition can be satisfied by choosing the orientation of $\mathbf{g}$ with respect to $\mathbf{e}_z$ appropriately.

In contrast with Ref. 1, the nonlinear electric displacement in question here cannot be derived by varying some expression for the free energy (or the Lagrangian) with respect to the field E. For this reason, the general conclusion (reached by Zel'dovich[1]) that translational invariance implies momentum flux conservation applies only to systems having a Lagrangian—and not to the case we are discussing in the present paper. At the same time, it follows that the nonlinearity which we are discussing here can be caused only by processes involving the absorption of light. This absorption may be so weak that we can ignore the change in the radiant energy over the entire length of the sample, but it is of fundamental importance for giving the nonlinearity the appropriate tensor structure, which permits a four-wave energy exchange between oppositely directed waves, with a transfer of momentum flux from the radiation to the medium.

In the case $\zeta = \pm 1$, i.e., when the wave E_L is circularly polarized, l becomes purely imaginary, and the gain vanishes. In this case we have only a nonlinear phase shift of E_{s^+} (if $\zeta = 1$) or of E_{s^-} (if $\zeta = -1$) of the signal wave. We also note that a nonlinear phase shift of the strong wave also occurs and gives rise to a self-focusing or self-defocusing, depending on the sign of the eccentricity of the wave and on its propagation direction with respect to the magnetic field.

In summary, this letter predicts a new parametric interaction between light waves. This interaction may find some interesting applications, e.g., for optical phase conjugation.[3]

We will not go into the specific mechanisms by which the light affects the gyration vector, but we do wish to point out that nonuniform heating of ferromagnets by interference fields seems promising. In this case we would have $\eta \propto \partial g/\partial T \propto (T_i - T)^{-1/2}$, and η would rise sharply toward the Curie point T_C.

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[2]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred. (Electrodynamics of Continuous Media, Addison-Wesley, Reading, Mass, 1960).
[3]Ba. Ya. Zel'dovich, N. F. Pilipetskiĭ, and V. V. Shkunov, Usp. Fiz. Nauk **138**, 249 (1982) [Sov. Phys. Usp. (to be published)].

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