Reduction of the U(∞) supersymmetry theory to a random-matrix model

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The limit $N \to \infty$ is analyzed for U(N)-invariant supersymmetry theories. It is shown that it is not necessary to freeze the momenta when such theories are reduced to a random-matrix model by the Eguchi-Kawai method. This assertion significantly simplifies numerical calculations in such theories.

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The paper of Eguchi and Kawai[1] has attracted considerable interest in the limit $N \to \infty$ of U(N)-invariant gauge and general matrix theories.[2–5] The basic advantage of this new approach to the limit $N \to \infty$ is that in the first approximation in N any invariant quantity can be written as an average in a suitable model of random matrices which do not depend on the coordinates. For the matrix ϕ^4 theory, for example, the quantity $\langle (1/N)\,\mathrm{Tr}\phi(x)\phi(0)\rangle$ is given in the limit $N \to \infty$ by

$$\int dp < \exp(-S(p,\ \phi))\ \frac{1}{N}\ Tr\ (e^{-ipx}\ \phi\, e^{ipx}\ \phi)>_{\phi},$$

$$S(p,\ \phi) = Tr\ (-\frac{1}{2}[\,p_m, \phi\,]^2 + \frac{1}{2} m^2 \phi^2 + \frac{g}{N}\ \phi^4\,).$$

Here ϕ is an $N \times N$ matrix, the p_m are diagonal $N \times N$ matrices ("momenta")[1)], and $\langle \cdots \rangle_{\Phi}$ denotes an average over ϕ, i.e.,

$$<\ldots>_{\phi} = \int d\,\phi\ \exp(-S(p,\phi))\ \ldots\ /\ \int d\,\phi \exp(-S(p,\phi)).$$

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This procedure of first taking an average over ϕ and then integrating over momentum has been termed "momentum freezing."

Our assertion is that momentum freezing is not necessary in the limit $N \to \infty$ in supersymmetry theories; it is sufficient to simply integrate over the superfield ϕ and over p. (Strictly speaking, we will prove this assertion only for nongauge theories; see the text below regarding the Yang-Mills supersymmetry theory.)

This assertion is based on the circumstance that the factor $Z(p) = \int d\phi \exp(-S(p,\phi))$, by which the integral over the fields of ϕ is divided, is equal to one in supersymmetry theories. Working from this fact, integrating over momenta, and letting N go to infinity, we find the well-known assertion that the free energy vanishes in supersymmetry theories.[6] We turn now to some exact formulations.

For definiteness, we consider the Wess-Zumino matrix model. We denote by $\phi(\theta, \bar{\theta})$ a chiral superfield. Here "superfield" means that $\phi(\theta, \bar{\theta})$ transforms under supersymmetry transformations in accordance with the following generators, which realize a supersymmetry algebra in the space of $N \times N$ matrices that depend on θ, $\bar{\theta}$:

$$S_a = \frac{\partial}{\partial \theta^a} - (\sigma^m \bar{\theta})_a [p_m, \qquad \bar{S}_{\dot{a}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} + (\theta \sigma_m)_{\dot{a}} [p_m .$$

Chirality of the superfield $\phi(\theta, \bar{\theta})$ means that $D_{\dot{a}} \phi(\theta, \bar{\theta}) = 0$, where $\bar{D}_{\dot{a}} = -\partial/\partial\bar{\theta}^{\dot{a}} - (\theta\sigma_m)_{\dot{a}} [p_m$. The action for the reduced form of the matrix Wess-Zumino model is

$$S(p, \phi) = \int d^2\theta\, d^2\bar{\theta}\, Tr\, \bar{\phi}\, \phi - \frac{m}{2}(\int d^2\theta\, Tr\, \phi^2 + \text{э.c.}) - \frac{g}{N}(Tr L_{int} + \text{H.a.}),$$

$$L_{int} = \int d^2\theta\, \phi^3.$$

The proof that $Z(p) = 1$ is analogous to the proof that the partition function is unity in supersymmetry theories,[6] since here again the Lagrangian $L_{\rm int}$ is an F-component of a supermultiplet. It accordingly appears upon supersymmetry transformations over the ψ component of this supermultiplet:

$$\delta(Tr\, \psi) = \epsilon\, Tr L_{int} + (\sigma_m \bar{\epsilon})\, Tr [p_m, A],$$

where we have also introduced the other components of the supermultiplet to which $L_{\rm int}$ belongs. After an integration over $\phi(\theta,\bar{\theta})$ the left side vanishes by virtue of the supersymmetry, while the last term on the right vanishes by virtue of Tr; the first term on the right is therefore also equal to zero. This result means that the derivative of the partition function with respect to the coupling constant vanishes; i.e., it becomes equal to the partition function for a zero coupling constant. The latter is one, since the determinants for the fermions and bosons cancel out. This result, after an integration over p, corresponds to a cancellation of the zero-point vibration energies of the fermions and bosons. We wish to emphasize that all these cancellations occur even before the limit $N \to \infty$, although the reduced model is, of course, equivalent to the unreduced model only in the limit $N \to \infty$.

For the Yang-Mills supersymetry theory (we mean the uninflated supersymmetry) our assertion has the consequence that the limit $N \to \infty$ of this theory is described by an action which is the ordinary action of the Yang-Mills supersymmetry theory, if the gauge superfield $V(x,\theta,\bar{\theta})$ in it is made independent of x and if the integration over dx is omitted. We wish to emphasize that momenta are not introduced at the outset in this approach, in contrast with the case of a purely gauge theory[2-5]; they arise during the construction of the perturbation theory, as parameters describing a manifold of classical vacuums (in this case, this manifold is not $V=0$ with an accuracy to gauge transformations, and V is a diagonal matrix with a zero D component), and the momenta are the vector component of this theory: $p_m \theta\sigma_m \bar{\theta}$. It can be shown that such a theory satisfies a supersymmetry contour equation.[7] So far, however, we have not been able to construct a perturbation theory in an explicitly supersymmetry form. It should be noted in this connection that for the ordinary Yang-Mills theory the corresponding reduced model does not have an explicit Lorentz invariance[2]; this invariance is restored only in the limit $N \to \infty$.

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[1]Because the matrix elements p_m give rise to momenta in the Feynman diagrams in the derivation of a perturbation theory.

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