Real part of the forward elastic scattering amplitude and high-energy behavior of the total cross sections

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Dispersion-relation calculations show that data obtained on the ratio of the real and imaginary parts of the forward elastic scattering amplitude at the SPS colliding-beam installation will simplify extrapolation of the total *pp* cross sections to ultrahigh energies.

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Information on the behavior of the total cross sections for high-energy particle interactions is very important for deriving a theory of strong interactions and also for studying the passage of high-energy particles through matter. The direct information obtained from cosmic rays is afflicted by a comparatively large error. So far, the extrapolations of the total cross sections, which have been proposed on the basis of various theoretical arguments, cannot be judged reliable, as can be seen from the fact that many of them have had to be discarded as accelerator energies have moved upward and also from the fact that there are several distinct extrapolation formulas which are quite successful in reproducing the behavior of the cross sections up to ISR energies but which predict different types of behavior at higher energies.

Let us examine how these formulas correspond to data on the total cross sections obtained at the SPS colliding-beam installation at $\sqrt{s} = 540$ GeV, where $\sigma_t = 66 \pm 7$ mb. We will show what these formulas predict about the ratio of the real and imaginary parts of the elastic amplitude at these (and higher) energies if dispersion relations are used. We will see that when this ratio is measured at the SPS it will become possible to choose more accurately among the various proposed extrapolations.

We have selected several extrapolations of the total cross sections which were published before SPS results at $\sqrt{s} = 540$ GeV became known. We should point out that the cross sections in closest agreement with experiment were predicted in papers based on certain definite theoretical arguments.

Extrapolation 1 (Ref. 1) is based on the Regge quark model of a supercritical $[\alpha_P(0) > 1]$ Pomeron and describes the following increase in the total cross section with the energy:

$$\sigma_t = 4,40 N_q (s/2)^{0,13} + 7,83 (N_{\bar{d}} + 2N_{\bar{u}})(s/2)^{-0,50} + 4,00 N_q N_{NS} (s/2)^{-0,2}, \tag{1}$$

where N_q, $N_{\bar{d}}$, $N_{\bar{u}}$, and N_{NS} are the total number of quarks and the numbers of d, $\bar{u}$, and nonstrange quarks in the incident hadron; and $s = 2\,mE$, where E is the energy in the laboratory frame, and m is the proton mass. Although expression (1) violates the Froissart limit, it is assumed that this is only a preasymptotic behavior and that screening [not incorporated in (1)] will eliminate the contradiction at very high energies. At $\sqrt{s} = 540$ GeV, we find $\sigma_t = 66.4$ mb from (1).

Extrapolation 2 (Ref. 2), which we took from a review of papers on the supercritical Pomeron, cannot be described by a simple analytic expression [it is the "(iii)" fit in Ref. 2]. At $\sqrt{s} = 540$ GeV, it leads to

$$\sigma_t = 62.3 \text{ mb}. \tag{2}$$

In the calculations we also used the three following, purely empirical formulas, which have been used previously to describe the total cross sections up to ISR energies.

Extrapolation 3 (Ref. 3) is used very frequently with a Froissart growth and power-law corrections:

$$\sigma_t = 41.9\,E^{-0.37} - 24.2\,E^{-0.55} + 27.0 + 0.17 \ln^2 s \,. \tag{3}$$

At $\sqrt{s} = 540$ GeV we find $\sigma_t = 54.4$ mb.

Extrapolation 4 (Ref. 4) has a Froissart growth and logarithmic corrections:

$$\sigma_t = 50.87 - 5.23 \ln s + 0.54 \ln^2 s\,. \tag{4}$$

Extrapolation 5 (Ref. 5) has the cross sections increasing logarithmically:

$$\sigma_t = 25.4 + 2.145 \ln s\,, \tag{5}$$

At $\sqrt{s} = 540$ GeV we find $\sigma_t = 52.4$ mb.

We have used these five extrapolation formulas[1)] to calculate the ratio (ρ) of the real and imaginary parts of the amplitude for forward elastic pp scattering, working from the customary dispersion relations:

$$\rho_{pp}(E) = \frac{c}{E\sigma_t^{pp}(E)} + \frac{1}{\pi\,\sigma_t^{pp}(E)} \int_m^\infty dE' \left\{ \frac{\sigma_t^{pp}(E')}{E' - E} - \frac{\sigma_t^{p\bar{p}}(E')}{E' + E} \right\}, \tag{6}$$

where the proton-antiproton cross section is assumed to be related to the pp cross section by an expression of the type $\sigma_t^{p\bar{p}} = \sigma_t^{pp} + AE^{a-1}$ (the values $A \approx 48$ and $a \approx 0.45$ are experimental[6]; the constant C was determined from the vanishing of ρ at $E = 315$ GeV).

The results are shown in Fig. 1. The circles on the curves corresponding to the five different extrapolations of the total cross section mark the point $\sqrt{s} = 540$ GeV, and the asterisks mark the UNK energy, $\sqrt{s} = 6 \times 10^3$ GeV.

We can draw several conclusions from this figure. 1—At the SPS energies, with ρ measurements accurate to something on the order of 0.005–0.01, we can reliably distinguish the individual curves (except 2 and 3). 2—The approximate equality of the cross sections at the same energies in the different extrapolations by no means implies

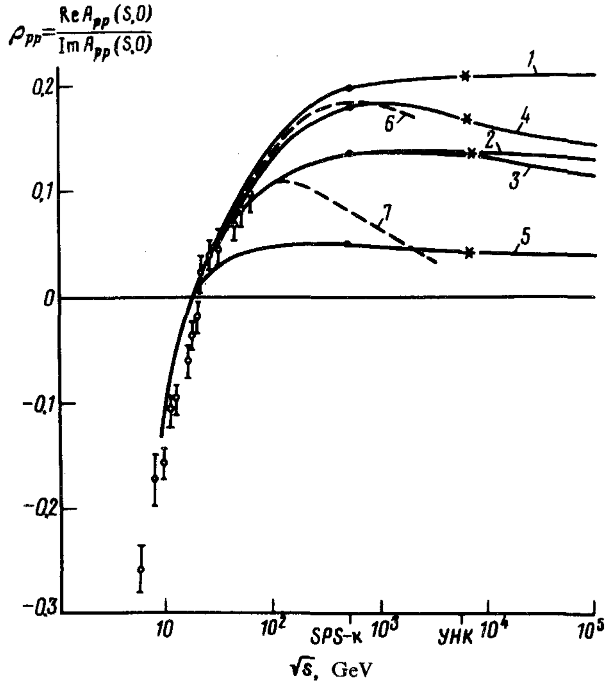

FIG. 1.

that the corresponding extrapolations yield approximately equal values of ρ at these energies (curve 1 is farther from curve 2 than from curve 4, while extrapolations 2 and 3 yield identical predictions for ρ). Therefore, the assertion of a local relationship between ρ and σ is not correct at the present energies. 3—The logarithmic growth of the cross sections is not satisfactory according to either the ISR data on ρ or the total cross sections from the SPS. 4—Over the energy range from SPS energies up to the planned UNK colliding-beam installation at Serpukhov ($\sqrt{s} = 6\times 10^3$ GeV) the ratio ρ should remain essentially constant. 5—Theoretical extrapolations 1 and 2 predict a weaker energy dependence for ρ than that predicted by the empirical extrapolations, (3)–(5). In the latter extrapolations, ρ falls off slightly with increasing energy after going through a maximum. 6—The ρ maximum is more noticeable if there are negative contributions to the cross section (extrapolations 3 and 4).

We might note that these conclusions hold if all the coefficients in these extrapolations are assumed to be absolutely accurate. However, slight errors in the determination of these coefficients can give rise to a corresponding "band of curves" in the figure. Furthermore, a deviation of the actual cross section from the extrapolation curve at high energies may have an effect. For example, the band of ρ values obtained through specific choice of an analytic energy dependence of the amplitude in Ref. 7 (dashed curves 6 and 7 in Fig. 1) is very wide, although the only difference between the extrapolations represented by the upper and lower curves is that instead of exhibiting the Froissart growth [with the large coefficient of 0.65 instead of 0.54 in (4)!] the cross section exhibits a plateau at about 85 mb (curve 7). We nevertheless believe that experimental data on ρ from the SPS (along with more accurate measurements of the total cross section) will provide further arguments for choosing among the various possibilities. Although we have confined this discussion to pp scattering, the difference between p and $\bar{p}$ at these energies is so small as to be negligible, and the curves found here can also be applied to the $p\bar{p}$ interaction.

[1)]In all these formulas, S is in GeV^2, E in GeV, and σ_t in mb.

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