Topological model of a Heisenberg spin glass

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Two aspects of a model recently proposed for a Heisenberg spin glass are discussed: the asymptotic freedom of the theory and the absence of a long-range order with a correlation radius which vanishes exponentially with the temperature. The latter circumstance means that $d = 3$ is the lower critical dimensionality of the model.

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Several phenomenological theories for spin glasses proposed in recent years are based on the representation of gauge invariance[2–4] (see also Refs. 5 and 6). Obukhov and the present author[1] recently managed to solve the microscopic XY model—and of a spin glass—the so-called Villen model—and showed that near the point of a so-called topological phase transition (at which frustration loops are broken, and frustration lines of infinite length appear) the microscopic Villen theory on the lattice actually transforms into the standard gauge field theory [with an SO(2) Abelian gauge group] with Higgs bosons (see Ref. 7, for example). It is not possible, however, to derive phenomenological gauge theories with a non-Abelian SO(3) group[2–6] from a microscopic Heisenberg Hamiltonian with random couplings. In Ref. 1 we proposed a simple-minded generalization of the results of the XY model to the case of Heisenberg spins; this generalization is a standard SO(3) gauge theory with a vector Higgs boson (aside from a minor complication stemming from the need to work by the replica method). Although clearly of a phenomenological nature, this theory has several amusing physical aspects.

1. Like any non-Abelian gauge theory, it has asymptotic freedom. Asymptotic freedom itself has essentially never been encountered in solid state physics,[1)] so that its existence should be regarded as essentially a basic assumption of the theory.[1] It should be kept in mind that so far we have nothing approaching reliable theoretical results in this field, so that everything written below is of a crude qualitative nature.

Above the topological transition point, where the frustration lines are closed, i.e., where the Higgs mechanism has not yet operated, the gauge fields are massless, and asymptotic freedom means an infrared catastrophe. At this point, we cannot say much about this phase. I would like to point out another possibility, which assigns a mass to all the gauge bosons, thereby eliminating the infrared catastrophe. This possibility is the spontaneous appearance of an average constant but nontrivial gauge field A_{ok} [we are using the standard[1–7] notation A^{α}_{k}, where α, β,... and k, l,... are respectively the isotopic (spin) and spatial indices] which gives rise to a constant field (curvature)

$$\mathbf{F}_{okl} = (\mathbf{A}_{ok} \times \mathbf{A}_{ol}). \tag{1}$$

Clearly, the finite (constant) $\mathbf{F}_{kl}$ means in principle the appearance of a finite density of

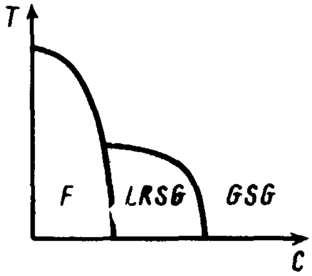

FIG. 1.

disclinations (currents)[1–7]:

$$j_0^k \sim D_l \mathbf{F}^{kl} \sim (\mathbf{A}_{ol} \times \mathbf{F}_0^{kl}), \tag{2}$$

where D_k is the covariant derivative.[1–7]

Choosing (for example) A^{α}_{ok} in the form $(F_0)^{1/2}\delta^{\alpha}_k$ or

$$F_{okl} = F_0 e_{kla}, \tag{3}$$

we find a state with a long-range order of the type discussed by Halperin and Saslow[9] and Andreev.[10] A transition to a spin-glass state with a long-range order (LRSG) from a ferromagnetic state (F) occurs in Fig. 1 as couplings of the wrong sign are added (cf. Fig. 4 in Ref. 1). Equations (2) and (3) give the source of the Halperin-Saslow-Andreev order parameter in terms of the density of disclinations (or gauge fields).

2. After the frustration lines (disclinations) of infinite length appear, the Higgs mechanism operates, sooner or later, and the system becomes a genuine spin glass[1] (GSG). At zero temperature, one of the gauge bosons [an SO(2) photon][7] and some of the Higgs bosons (see the corresponding equation in Section 4 in Ref. 1) remain massless. At a nonzero temperature, however, the phase correlation radius becomes finite. The screening of photons and Goldstone particles is caused by a Polyakov-'t Hooft plasma of monopoles[11] (see also Ref. 12), which are present in this model.[6] These monopoles are of course not present at zero temperature. At a low temperature their density is exponentially small, and the corresponding Debye length is[2)]

$$R_c \sim \exp(\text{const}/T). \tag{4}$$

Equation (4) shows that the GSG phase is probably paramagnetic (or superparamagnetic), as in the XY model.[1]

The exponential expression in (4) and the same expression derived previously in the XY model[1] confirm Anderson and Pond's idea[13] that $d = 3$ is a lower critical dimensionality in a spin glass with a continuous symmetry (see also Ref. 2).

[1)] It is interesting to note that Feĭgel'man and Tsvelik[8] were the first to run into the asymptotic-freedom situation in the theory of spin glasses.

[2)] In Ref. 1 we forgot about Polyakov—'t Hooft monopoles and incorrectly asserted that the correlation radius was infinite in the GSG phase at finite T.

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