Quantum effects in the conductivity of two-dimensional disordered systems

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It is shown that the coefficients multiplying the logarithms in the expressions for the quantum corrections to the conductivity of two-dimensional systems depend on the characteristics of the scattering potentials and the geometry of the Fermi surface, and also on the energy of the carriers.

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The quantum corrections to the conductivity of two-dimensional disordered systems have attracted considerable recent interest. It has been shown[1,2] that the correction to the conductivity due to localization effects is of the form

$$\sigma_L = \frac{e^2}{2\pi^2\hbar}\,\alpha \ln \frac{\tau}{\tau_{in}}\,. \tag{1}$$

Here τ is the free travel time, τ_{in}^{-1} is the inelastic collision frequency, and $\alpha = 1$ in the model which is ordinarily used.[1,2]

Logarithmic corrections to the conductivity also arise when the electron-electron interaction is taken into account[3]:

$$\sigma_I = \frac{e^2}{2\pi^2\hbar}\beta \ln\frac{T\tau}{\hbar}\ , \tag{2}$$

where T is the temperature, $\beta = 1 - F$, $F = \langle V_e(\mathbf{p} - \mathbf{p}')\rangle\ V_e^{-1}(0)$, $V_e(\mathbf{p})$ is the Fourier transform of the interaction potential of the carriers, the momenta $\mathbf{p}$ and $\mathbf{p}'$ lie on the Fermi surface, and the angle brackets denote an average over the Fermi surface.

Expressions (1) and (2) are obtained under two simplifying assumptions: a) that the potential for scattering by impurities has a short-range nature; b) that the Fermi surface is isotropic. For the two-dimensional systems of practical interest, however, these assumptions are often incorrect.

In this paper the quantum corrections to the conductivity are calculated with an anisotropic dispersion relation for the carriers and with allowance for a finite range of the scattering potentials. In this generalization model the logarithmic dependence on τ_{in} and T in expressions (1) and (2) is preserved, but the quantities α and β generally become tensors which depend on both the characteristics of the scattering centers and the geometry of the Fermi surface. Such a dependence was noted by Bhatt and Ramakrishnan,[4] but the model they used is unrealistic.

Let us assume for simplicity that the Fermi surface is simply connected and convex.The sum of the fan diagrams K (the wavy line in Fig. 1), which describes diffusion in the particle-particle channel,[2] satisfies the integral equation illustrated in Fig. 1. The solution of this equation can be found in the form of a series in powers of the momentum transfer $\mathbf{q}$. The leading singularity with respect to $\mathbf{q}$ is a term of the form (see Ref. 5)

$$K(\mathbf{p}, \mathbf{p}', \mathbf{q}, \omega) = \frac{2\Gamma(\mathbf{p})\Gamma(\mathbf{p}')}{\pi\hbar\rho_F}\ \frac{1}{i\omega + D_{ij}q_iq_j}\ . \tag{3}$$

Here Γ is the single-particle damping, ρ_F is the density of states on the Fermi surface, ω is the frequency,

$$D_{ij} = \frac{\hbar}{2\pi\rho_F}\int d\varphi\, P(\varphi)Q_i(\varphi)V_j(\varphi)$$

is the diffusion tensor, which is related to the Boltzmann conductivity by the Einstein relations, V_j are the components of the carrier velocity, and

$$P(\varphi) = \frac{1}{2\pi}\left|\frac{D(k_x, k_y)}{D(\epsilon, \varphi)}\right|_{\epsilon = \epsilon_F}$$

is the Jacobian of the transformation from k_x and k_y to the energy and angle variables ϵ and φ. The function Q_i are given by the integral equations

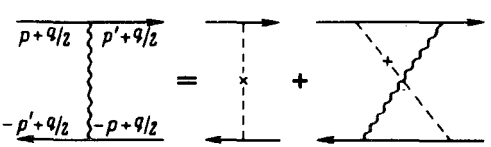

FIG. 1.

$$\int d\varphi' P(\varphi') W(\varphi, \varphi')(Q_i(\varphi) - Q_i(\varphi')) = V_i(\varphi) . \quad (4)$$

Here W is the interaction line, which in the first Born approximation is given by $W(\mathbf{p}, \mathbf{p}') = n|u(\mathbf{p}, \mathbf{p}')|^2$, where n is the impurity concentration, and $u(\mathbf{p}, \mathbf{p}')$ is the matrix element of the scattering potential.

Because of the inexact nature of the scattering potentials, one must also dress the vector vertices with impurities.[6] The dressed vertices are associated with expressions $e\Gamma(\varphi)Q_i(\varphi)$. Finally, the localization correction to the conductivity in our model is given by an expression of the form in (1), but where α is the tensor

$$\alpha_{ij} = \tilde{D}_{ij} D_0^{-1}. \quad (5)$$

Here $D_0 = (\det D_{ij})^{1/2}$ and

$$\tilde{D}_{ij} = \int d\varphi P(\varphi)\Gamma(\varphi)Q_i(\varphi)Q_j(\varphi) .$$

In evaluating the corrections to the conductivity that are due to the interaction, we take into account the same diagrams that were considered by Al'tshuler *et al.*[3,7] Using Eq. (3) and taking the dressing of the vector vertices into account, we obtain an expression for the quantity β in (2):

$$\beta_{ij} = D_{ij} D_0^{-1} - F\alpha_{ij} . \quad (6)$$

Let us consider the various limiting cases of Eqs. (5) and (6).

1) Point scattering centers. In this case Eqs. (5) and (6) give the results of Ref. 8 for anisotropic systems with point impurities:

$$\alpha_{ij} = D_{ij} D_0^{-1}, \quad \beta_{ij} = \alpha_{ij}(1 - F).$$

2) Circular Fermi surface and axially symmetric potentials. Here α and β are given by

$$\alpha_{ij} = \delta_{ij}\sigma_s/\sigma_t, \quad \beta_{ij} = \delta_{ij}(1 - F^*), \quad (7)$$

where σ_s is the cross section for scattering by impurities, σ_t is the transport cross section, and $F^* = F\sigma_s/\sigma_t$.

The ratio σ_s/σ_t is greater than or equal to unity and increases with the energy of the carriers. The qualitative form of the dependence of α and β on the Fermi energy is given in Fig. 2. It is seen that at high enough energies, F^* can become greater than one (for $F < 1$), i.e., inversion of the sign of the correction σ_I can occur. We note that the

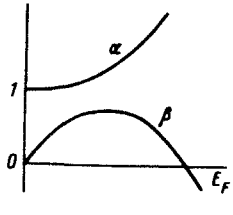

FIG. 2.

value $F^* > 1$ observed experimentally by Bishop *et al.*[9] cannot be explained in terms of a point-center model.

The results of this study show that the coefficients multiplying the logarithm in the expressions for the quantum corrections to the conductivity of two-dimensional systems can change continuously as the Fermi energy increases. An experimental check of this dependence would be of considerable interest.

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