Bremsstrahlung of electrons in an atomic potential: the classical character of the spectrum; scaling laws

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A simple classical theory is developed for the bremsstrahlung of kilovolt electrons in a Thomas-Fermi potential, permitting the universalization of the voluminous body of numerical quantum results for this problem, which, in turn, have been confirmed by recent experiments.

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1. *Introduction*. The problem of finding the bremsstrahlung spectrum of electrons on a many-electron atom is solved in practice only in the approximation of a static

atomic potential, the applicability of which is still insufficiently studied (see Ref. 1 and the literature cited therein). Recent experiments[2] on the differential (with respect to the photon departure angle θ) bremsstrahlung cross section for incident electron energies E in the keV range are in very good agreement with the results of the corresponding numerical quantum calculations,[3] thereby demonstrating the applicability, for the important frequencies in the integrated bremsstrahlung spectrum, of the static model of the atom.

For this reason the theory of bremsstrahlung in a static atomic potential acquires significantly greater practical interest, so that for the problem of interest here—the bremsstrahlung spectrum integrated over θ—the existing, numerical level of description[4,5] is inadequate and should be replaced, to the extent possible, by a physically satisfying analytical description. It turns out that this can be accomplished precisely for the keV range of electron energies in a simple classical framework based on the Thomas-Fermi model of the atom. The scaling laws thereby obtained permit one to universalize the voluminous body of the corresponding numerical quantum results.[4,5]

2. *The classical character of the spectrum*. By obtaining and studying the quantum corrections to the classical limit of the matrix element for a radiative transition in the continuum for a central potential,[6] one sees that the parameter measuring the quasi-classical character of the integrated bremsstrahlung spectrum of electrons on the Thomas-Fermi atom (qualitatively analogous to the parameter $Ze^2/\hbar v$ for bremsstrahlung in the Coulomb case[7]) is the quantity $1/\epsilon \sim Z^{4/3}/E_{\text{a.u.}}$, so that (in the framework of the static model of the atom) the spectrum becomes more classical as ϵ decreases.

This conclusion is in good agreement with the body of numerical quantum calculations[4,5] for neutral atoms: The corresponding spectral Gaunt factors for $\epsilon \lesssim 1$ and $Z \gtrsim 20$ are, to within 10–20%, functions of only the "natural" variables of the classical spectrum (ω is the radiation frequency, and Z the atomic number):

$$\epsilon = \frac{b\hbar^2}{m e^4}\frac{E}{Z^{4/3}} \approx 32.6\,\frac{E\,(\text{keV})}{Z^{4/3}}, \qquad \Omega = \sqrt{\frac{b^3}{2}}\,\frac{\hbar^3}{m e^4}\,\frac{\omega}{Z}, \quad b = 0.885, \tag{1}$$

(here $\hbar$ enters only as a parameter of the specified Thomas-Fermi potential); as ϵ decreases the degree of classical parametrizability improves, while for $\epsilon \gg 1$ it breaks down.

We note that for the bremsstrahlung of electrons on ions, by virtue of the high Coulombic character of the potential (compared to that of a neutral atom), the deviation of the spectrum from the classical will be smaller still.

3. *The "transport" limit, the "rotational" approximation, and the bremsstrahlung spectra*. In accordance with part 2, it is appropriate to construct a classical description of the bremsstrahlung spectra. A complete description, which, though classical, involves an extremely awkward numerical calculation, can be avoided by examining two limiting cases which are amenable to analytical treatment—the "transport" limit ($\omega = 0$) and the high-frequency approximation—which, as it turns out, are sufficient to describe the entire spectrum with acceptable accuracy.

For $\omega = 0$, the Gaunt factor, expressed in the familiar way in terms of the transport cross section for elastic scattering, is evaluated analytically in an appropriate way:

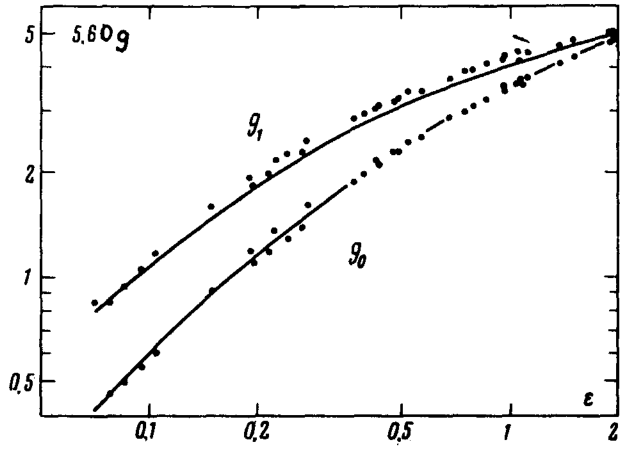

FIG. 1. The universal functions $g_0(\epsilon)$ and $g_1(\epsilon)$, shown by the curves, and, for comparison, the (recast) results of the numerical calculations of Lee *et al.*[4]

for $\epsilon \lesssim 10^{-2}$ one has $g_0 = 4.34\epsilon^{4/3}$, for $\epsilon \gg 1$ one has $g_0 = (\sqrt{3}/\pi)\ln 2.1\epsilon$, and for $10^{-2} \ll \epsilon \lesssim 1$ one has $g_0 = (4\sqrt{3}/\pi)\epsilon^2\bar{x}^2(\epsilon)$, where $\chi(\bar{x}) = \epsilon\bar{x}$ is a universal Thomas-Fermi function[8] (an exact tabulation of g_0 is in Fig. 1).

A simplification for large ω is achieved by constructing a new, "rotational" approximation[6] based on the existence of an approximate single-valued relation between the radiated bremsstrahlung frequency and the angular rotational frequency $\omega_{\rm rot}$ of the electron at its point of closest approach (r_0) to the center of the field. The corresponding delta function $\delta(\omega - \omega_{\rm rot}(r))$ introduced in the integral[7] $\int (r dU/dr)^2 \sqrt{1 - U/E}\, dr$, which is proportional to the spectrum-integrated stopping loss, enables one to find the bremsstrahlung spectrum. For the case of a neutral Thomas-Fermi atom we obtain the Gaunt factor

$$g_{\rm rot}(\Omega, \epsilon) = 3(\chi - y\chi')^2 \left[2 + \frac{\chi - y\chi'}{\chi + \epsilon y} \right]^{-1}, \quad \Omega \gtrsim \tilde{\Omega} \sim \sqrt{\epsilon}/\bar{x}(\epsilon), \tag{2}$$

where $\chi = \chi(y)$ and $y \equiv y(\Omega, \epsilon)$ is a root of the equation $\chi + \epsilon y = \Omega^2 y^3$.

It can be shown that for the region $\epsilon \leqslant 2$ the simplified lower (with respect to Ω) limit of applicability of formula (2) is the straight line $\Omega = 2\epsilon$, so that the corresponding universal function $g_{\rm rot}(2\epsilon, \epsilon) \equiv g_1(\epsilon)$ (Fig. 1) can be used, in combination with $g_0(\epsilon)$, to construct the simplest interpolation of the spectrum[1)]:

$$g(\Omega, \epsilon) = g_0(\epsilon) + \frac{\Omega}{2\epsilon}[g_1(\epsilon) - g_0(\epsilon)]. \tag{3}$$

The spectra obtained in this way are in good agreement with the results of Refs. 4 and 5 (Fig. 2; we note that in Ref. 4 a Hartree-Fock potential, rather than a Thomas-Fermi potential, was used).

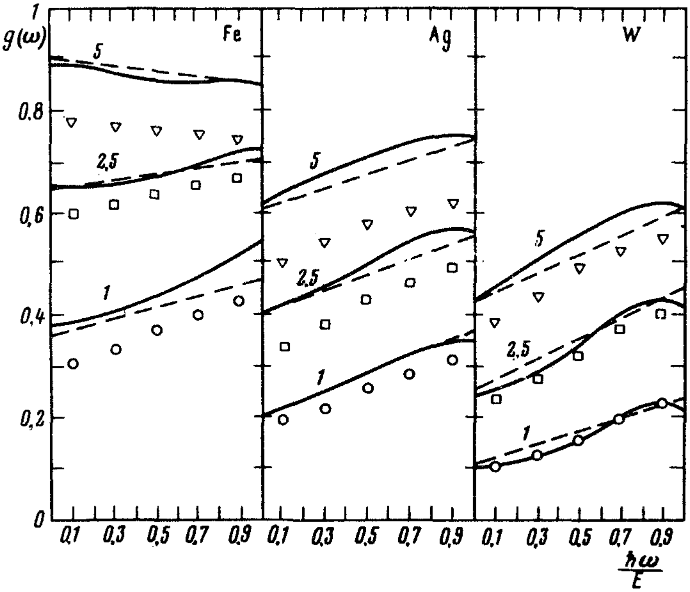

FIG. 2. The bremsstrahlung spectra of electrons on neutral atoms of Fe, Ag, and W as calculated by formula (3) (dashed lines), compared with the results of Lee *et al.*[4] (the curves) and Zhdanov[5] (the circles, squares, and triangles). The curves are labeled by the electron energy in keV.

4. *Scaling laws.* It follows from parts 2 and 3 that the natural variables of the classical spectrum, ϵ and Ω, are approximate scaling parameters for the initially quantum-mechanical bremsstrahlung spectrum, which is richer in parameters. This circumstance permits one to universalize the spectra of Refs. 4 and 5 into a single family of curves $g(\Omega,\epsilon)$ by recasting them in variables Ω and ϵ. By virtue of the role of the "reference" functions $g_0(\epsilon)$ and $g_1(\epsilon)$ in the description of $g(\Omega,\epsilon)$, which was established in part 3, a general comparison of the quantum results with the classical results can be made at the level of these functions. It is more convenient for this purpose to use the data of Ref. 4, in view of the lack of points for $\omega = 0$ in Ref. 5 (see Fig. 1).

Our analytical description of the bremsstrahlung spectrum enables us to discover still another scaling law, for the short-wavelength boundary of the spectra, $\omega_{\max} = E/\hbar$. Specifically, owing to the weak dependence of $g_{\rm rot}$ on ϵ in the region $\{\Omega \sim 2\epsilon, \epsilon \leqslant 2\}$ (which is precisely the region in which $\omega_{\max}$ falls), for points on the spectra with $\hbar\omega/E = \text{const} \simeq 1$ the Gaunt factor is a function of E/Z alone. This property is well illustrated by an analysis of the results of Ref. 5 for the maximum frequency $\omega = 0.9\omega_{\max}$ calculated in that paper, and also by the corresponding results of Ref. 4 (Fig. 3).

It follows from all we have said that for $\epsilon \leqslant 2$ and $Z \gtrsim 20$ the bremsstrahlung spectra go over smoothly between the two scaling laws $G_0(E/Z^{4/3})$ and $G_{\max}(E/Z)$ as the frequency changes from $\omega = 0$ to $\omega = \omega_{\max}$.

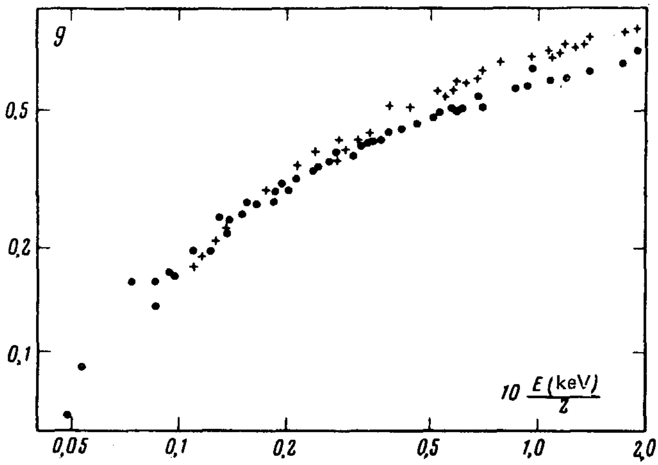

FIG. 3. A replotting of the results of the numerical calculations of Lee *et al.*[4] (the crosses) and Zhdanov[5] (the points) for $\omega = 0.9\omega_{max}$, illustrating the scaling law for the short-wavelength boundary of the spectrum.

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[1)] Formula (3) evidently answers the question which frequently arises as to the "effective" charge $Z^2_{eff}(\omega,E,Z)$ associated with bremsstrahlung of frequency ω.

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