Neutrino oscillations in grand unified models with a horizontal symmetry

Z. G. Berezhiani and Dzh. L. Chkareuli
Institute of Physics, Academy of Sciences of the Georgian SSR

(Submitted 29 January 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 6, 285–288 (20 March 1983)

A new mechanism for the generation of neutrino masses is discussed in the SU(5) grand unified model with a horizontal symmetry $SU(3)_H$ between quark-lepton generations. A strong $\nu_\mu - \nu_\tau$ mixing is predicted in a model with three quark generations. The primary oscillation effects in the six-generation model involve the mixing of only left-hand and right-hand neutrinos.

PACS numbers: 12.10.En, 11.30.Ly, 12.35.Kw, 14.60.Gh

1. We believe[3] that the hypothesis of a local, spontaneously violated horizontal symmetry[1,2] $SU(3)_H$ will lead to a better understanding of the quark-lepton mass spectrum in the grand unified models which are expansions of the Georgi-Glashow SU(5) model.[4] The particular structure of the matrix of vacuum expectation values of the horizontal symmetry, which is related to the simple multiplets of scalar $SU(3)_H$ triplets ξ^α and η^α and the sextet $\chi\{^{\alpha\beta}\}$ $(\alpha,\beta = 1,2,3)$

$$\langle \xi^a \rangle = p\,\delta_{a1}, \quad \langle \eta^a \rangle = q\delta_{a3}, \quad \langle \chi^{\{a\,\beta\}} \rangle = \mathrm{diag}\,(r_1, r_2, r_3)^{a\,\beta}; \\ r_3 \sim bp \sim b^2 q \sim b^2 r_2 \cong b^6 r_1, \quad b = (m_c/m_u)^{1/4} \tag{1}$$

($m_{u,c}$ are the masses of the u and c quarks), leads to mass matrices for the quarks and leptons in agreement with experiment, as was shown in Ref. 3. The sole scale value for the violation of $SU(3)_H$ symmetry in this case is $r_3 \equiv V_H \sim V$, where V is the scale value in the SU(5) theory.[4]

Lyubimov *et al.*[5] have recently found a lower limit on the mass of the electron neutrino, ν_e: $m_{\nu_e} \geqslant 14$ eV. If the masses of the neutrinos increase from generation to generation on the average, as the quark masses do, in accordance with the horizontal hierarchy[3] $r_1{:}r_2{:}r_3$, we find a result in sharp contradiction of the cosmological restrictions[4] on the masses of the two other neutrinos, ν_μ and ν_τ. In this letter we show that according to the horizontal symmetry the neutrino masses should obey an "inverse" hierarchy,

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = \frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3}, \tag{2}$$

and we will compare our results on the neutrino oscillations for models with three and six quark-lepton generations (models *A* and *B*, respectively).

2. The fermion sector of the *A* model [the minimal expansion of the standard SU(5) model] is formed by the $SU(5) \otimes SU(3)_H$ multiplets

$$\psi^{ia}(\overline{5},\overline{3}),\quad \psi^{a}_{[ij]}(10,\overline{3}),\quad N^{(n)}_{a}(1,3) \tag{3}$$

(a left-hand helical "filling"), where we now have, in addition to the ordinary quarks and leptons of SU(5), the horizontal triplets of $SU(3)_H$ (ψ multiplets, $i,j = 1,\dots, 5$; $\alpha = 1,2,3$). We have introduced 15 fields $N^{(n)}_\alpha$ ($n = 1,\dots, 15$) to make the theory also free of anomalies in terms of the $SU(3)_H$ horizontal group. In the absence of these anomalies, there would be no point in introducing the $SU(3)_H$ symmetry.

On the other hand, the appearance of the fields $N^{(n)}_\alpha$ in the Lagrangian necessarily leads to the appearance of Majorana masses for the neutrino fields. This conclusion follows from the Yukawa couplings of the fields $\psi^{i\alpha}$ (which contains a neutrino) and $N^{(n)}_\alpha$ ($F_{nn'}$, and G_n are the Yukawa constants):

$$\Delta \mathcal{L} = F_{nn'} N^{(n)}_{\alpha} C N^{(n')}_{\beta} + G_n \psi^{ia} C N^{(n)}_{a} H_i , \tag{4}$$

where $\chi\{^{\alpha\beta}\}$ is the $SU(3)_H$ sextet, with which we are already familiar, and for which the vacuum expectation value is given by (1), and H_i is the standard scalar pentaplet of SU(5) (Ref. 4) ($\langle H_i\rangle = v\delta_{i5}, v \sim 100$ GeV). From the first coupling in (4) we find the masses of the fields N_α (for simplicity, we are considering only one of the fields $N^{(n)}_\alpha$, on the order of V_H with the horizontal hierarchy in (1):

$$m_{N_1} : m_{N_2} : m_{N_3} = r_1 : r_2 : r_3 . \tag{5}$$

From the second coupling, after the violation of the $SU(2) \otimes U(1)$ symmetry[4], a mixing of the fields N_α with the fields of neutrinos in the multiplet $\psi^{i\alpha}$ arises independently for each horizontal flavor ($\alpha = 1,2,3$). After diagonalization, we find (2) for the masses of the physical neutrinos: $m_{\nu_\alpha} \cong G^2 v^2/F r_\alpha, m_{\nu_1} \sim b^6 v^2/V_H \sim O(1\text{–}100)$ eV.

Since the mass matrices of the higher quarks (u,c,t) and neutrinos (ν_1,ν_2,ν_3) are diagonal in the $SU(5)\otimes SU(3)_H$ model,[3] the physical mixing angles of the quarks ($\sin\Theta_k \equiv s_k, k = 1,2,3$) and leptons ($\sin\Omega_k \equiv l_k$) follow from simply the matrices of the lower quarks (d,s,b) and the charged leptons[3] (e,μ,τ):

$$s_1 \cong \sqrt{m_d/m_s},\quad s_2^{(\pm)} \cong \frac{1}{2\sqrt{2}}\left[\frac{m_\mu}{m_\tau} \pm \frac{m_s}{m_b}\right]^{1/2},\quad s_3^{(\pm)} \cong \frac{m_s}{m_b} s_2^{(\pm)}, \tag{6a}$$

$$l_1 \cong \sqrt{m_e/m_\mu},\quad l_2^{(\pm)} \cong \frac{3}{2\sqrt{2}}\left[\frac{m_\mu}{m_\tau} \pm \frac{m_s}{m_b}\right]^{1/2},\quad l_3^{(\pm)} \cong \frac{m_\mu}{m_\tau} l_2^{(\pm)}, \tag{6b}$$

where $s^{(\pm)}_{2,3}$ and $l^{(\pm)}_{2,3}$ correspond to the two possible solutions for the CP phase δ: $\cos\delta \approx \pm 1$. The t-quark masses (in GeV)

$$m_t \approx 180 \ (\cos\delta \approx +1),\quad m_t \approx 580 \ (\cos\delta \approx -1), \tag{7}$$

which follow from the mass difference $K_L - K_S$ when we use our quark mixing angles, (6a), and take the quantum chromodynamics corrections[4] into account ($m_c = 1.4$ GeV), lead to an unambiguous choice of the solution with $\cos\delta \approx +1$. Values $m_t > 200$ GeV must be rejected[6]: They cause a rapid growth of the mass of the b quark, disrupting the basic mass relation of the SU(5) theory[4]: $m_b/m_\tau \approx 3$.

The mixing of the leptons [with $l_1 \cong 0.07$, $l_2^{(+)} \cong 0.3$, and $l_3^{(+)} \cong 0.006$; (6b)] gives rise to neutrino oscillations. For the normalized average intensities $(R_{rs} = \bar{I}_{\nu_r \nu_s} / I^0_{\nu_r \nu_r})$ we find (in percent)

$$R_{e\mu} \cong 1, \quad R_{e\tau} \cong 0,1, \quad R_{\mu\tau} \cong 16, \tag{8}$$

i.e., a rather strong $\nu_\mu - \nu_\tau$ mixing. What is the oscillation length for this transition? According to (2) and (1),

$$|m^2_{\nu_2} - m^2_{\nu_3}| \cong m^2_{\nu_2} \cong (m_u/m_c)^2\, m^2_{\nu_1}. \tag{9}$$

For the interval of m_{ν_1} values found in Ref. 5 we find the following for L, the oscillation length ($m_u \cong 4$ MeV, $m_c = 1.4$ GeV):

$$m_{\nu_1} = 14 \div 46 \text{ eV}\ ^5, \quad L = p\,(1500 \div 150) \text{ m}, \tag{10}$$

where p is the neutrino momentum, in MeV.

3. The fermion sector of model B follows from the fermion multiplet $\psi^a_{[bc]L} + \psi^a_{[bc]R}$ $(a,b,c, = 1,...,8)$ of SU(8) symmetry with constituent quarks and leptons.[2] The $SU(5) \otimes SU(3)_H$ fragments of this multiplet, in which we are interested here, are

$$\psi^{ia}_L, \quad \psi^a_{[ij]L}, \quad \psi_{\alpha L}; \quad \Psi^{ia}_R, \quad \Psi^a_{[ij]R}, \quad \Psi_{\alpha R} \tag{11}$$

$(i,j = 1,...,5,\ \alpha = 1,2,3)$, where the "left-hand" ψ multiplets contain the ordinary quarks and leptons (three generations), and the "right-hand" Ψ multiplets contain three new quark-lepton generations, with masses $\sim O(100)$ GeV and a $(V + A)$ weak-interaction structure. The horizontal triplets $\Psi_{\alpha L} \equiv N_{\alpha L}$ and $\Psi_{\alpha R} \equiv N_{\alpha R}$ acquire (independently) large $(\sim V_H)$ Majorana masses from couplings of the type in (4) with a horizontal hierarchy (5), and they induce the masses of the left-hand and right-hand neutrinos, respectively, according to (2). The quark and lepton generations in the multiplets ψ_L and Ψ_R are "constructed" from, respectively, left-hand and right-hand preons, which are mixed together weakly (with a nondiagonal mass ~ 1 eV; more on this below), so that the mass matrices of the ordinary quarks and leptons can be treated separately. In this case the sines of the quark and lepton mixing angles (S_k) and (E_k) are related to the values in (6) in the following way:

$$S_1 = s_1, \quad S_2^{(\pm)} \cong 3\, s_2^{(\mp)}, \; S_3^{(\pm)} \cong 3 s_3^{(\mp)}, \tag{12a}$$

$$E_1 = l_1, \quad E_2^{(\pm)} \cong \frac{1}{3}\, l_2^{(\mp)}, \; E_3^{(\pm)} \cong \frac{1}{3}\, l_3^{(\mp)}. \tag{12b}$$

In this case, again working from the mass difference $K_L - K_S$, we find

$$m_t \approx 28 \;\; (\cos\delta \approx +1), \qquad m_t \approx 16 \;\; (\cos\delta \approx -1), \tag{13}$$

and we choose the solution with $\cos\delta \approx +1$, in agreement with experiment ($m_t > 18$ GeV). It is easy to see from (12b) that in model B or the SU(8) model[2] the neutrino oscillation effects are suppressed with a change in the horizontal flavor [$E_2^{(+)} \cong 0.07$, $R_{\mu\tau} \cong 1\%$; cf. (8)].

In this model, however, there may be some strong oscillations of the left-hand neutrinos from the multiplet ψ and of right-hand neutrinos from the multiplet Ψ:

$\nu_L \to \nu'_R (\nu = \nu_e, \nu_\mu, \nu_\tau)$. As was mentioned in Ref. 7, in the SU(8) model[2] the Dirac masses of the neutrinos may be generated,

$$\mathcal{L}_m = m_0 (\bar{\nu}_{eL} \nu'_{eR} + \bar{\nu}_{\mu L} \nu'_{\mu R} + \bar{\nu}_{\tau L} \nu'_{\tau R}) , \tag{14}$$

by virtue of the Wheeler-Zel'dovich-Hocking gravitational mechanism,[7] which causes a transition of the left-hand SU(8) multiplet of constituent quarks and leptons, $\psi^a_{[bc]L}$, into the right-hand multiplets $\Psi^a_{[bc]R}$. For the preon confinement radius $R \sim 10^{-16}$ GeV^{-1} we find $O(1)$ eV (Ref. 7). The neutrino oscillations now depend on the ratio of the Dirac (m_0) and Majorana (m_ν and $m_{\nu'}, \nu = \nu_e, \nu_\mu, \nu_\tau$) neutrino masses. In all cases in which the condition $m_0 > (m_\nu, m_{\nu'})$ holds the mixing angles of the corresponding left-hand and right-hand neutrinos are $\phi_{\nu\nu'} \approx \pi/4$ (maximum mixing). In view of the hierarchy (2) for the Majorana masses, this condition seems feasible for $\nu_\mu(\nu'_\mu)$ or, especially, for $\nu_\tau(\nu'_\tau)$. Unfortunately, we are not able to find anything approaching an accurate estimate of the $\nu_{\mu L} \to \nu'_{\mu R}$ and $\nu_{\tau L} \to \nu'_{\tau R}$ oscillation lengths.

We wish to thank A. A. Ansel'm, O. V. Kancheli, and K. A. Ter-Martirosyan for useful discussions.

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Translated by Dave Parsons
Edited by S. J. Amoretty