# Generation of higher harmonics of an intense laser beam in a plasma

M. B. Isichenko and V. V. Yan'kov
*I. V. Kurchatov Institute of Atomic Energy, Moscow*

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The generation of higher harmonics of a pump beam by a mechanism involving a breaking of an electron wave is analyzed. The emission intensity is found to depend on the harmonic index in a power-law fashion, in agreement with experiment.

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Experiments on plasma heating by the beams from $CO_2$ lasers with power densities of $10^{14}$–$10^{15}$ W/cm$^2$ have revealed dozens of harmonics,[1–3] up to the 46th (Ref. 3). Theory for harmonic generation in the case of a slight nonlinearity has been derived quite thoroughly,[4,5] but the predicted emission intensities fall off exponentially with the harmonic index $n$, so that vanishingly low intensities are predicted at $n \cong 10$–$50$. We must therefore turn to the case of a finite nonlinearity. In this case the higher harmonics may be generated in a variety of discontinuous events, the simplest of which involves a breaking of an electron wave. The emission intensity depends on the harmonic index in a power-law fashion, with the exponent being determined entirely by the nature of the particular discontinuity. A well-known example is the transition radiation of a charged particle as it crosses a discontinuity in the dielectric function; this type of radiation falls off with the frequency as $\omega^{-4}$ (Ref. 6). In this letter we examine a more intense collective emission.

As a first example we consider the emission which occurs at the time of the breaking of a nonrelativistic electron wave in a homogeneous plasma. This is a quadrupole emission, since the net momentum transferred to ions in a homogeneous plasma is zero.[7] This breaking can occur at $\tilde{v} \gtrsim a\omega_0$, where $\tilde{v}$ is the velocity of the electrons caused by the pump beam, $a$ is the scale dimension of the wave inhomogeneity at the plasma boundary, and $\omega_0$ is the pump frequency. The motion of electrons near the breaking point may be treated in a zeroth approximation as free motion: $\mathbf{r} = \mathbf{r}_0(\mathbf{v}) + \mathbf{v}t$. Expanding $\mathbf{r}$ in a series in powers of $\mathbf{v}$, and noting that the first term vanishes at the time of the breaking, we find that a disk-shaped singularity arises at the breaking. The region in which $\mathbf{r}(\mathbf{v})$ becomes multivalued is described in cylindrical coordinates $(r,z)$ by the following inequality after a transformation to principal axes:

$$(r/R)^2 + (z/a)^{2/3} \leqslant t/t_0, \quad 0 < t \ll t_0 \equiv a/\tilde{v},$$

where $R$ is the scale transverse dimension of the current, and the time $t$ is reckoned from the beginning of the breaking. At the breaking point the electron density becomes infinite in accordance with

$$n_e \cong n_0 \, |t/t_0|^{-1}.$$

In the next approximation in $(\omega_{pe} t_0)^2$ the field produced by the electrons of this region, $E \cong 2\pi e n_0 a (t/t_0)^{1/2}$, causes the plasma electrons to radiate. We have the following equation for the change in the quadrupole moment of the electrons due to the breaking:

$$\dddot{D}(t) \cong e n_0 R^4 \omega_{pe}^2 \tilde{v} (t/t_0)^{3/2} \theta(t) .$$

Assuming that this singularity arises during each period of the laser pump beam, and expanding the resulting function in a Fourier series, we find the intensity of the emission from the plasma at the $n$th harmonic of the laser frequency:

$$W_n \cong \frac{m n_0 \tilde{v}^5}{c^5} \frac{R^8}{a^3} \frac{\omega_{pe}^6}{\omega_0^3} \frac{1}{n^5}, \quad n \gg \frac{1}{2\pi \omega_0 t_0} .$$

More intense dipole emission can occur in a collision of a broken electron wave with a perturbation of the ion density $n_i$. The change in the dipole moment of the electrons can be calculated from[7]

$$\ddot{\mathbf{d}}(t) = (e^2/m) \int \phi_e \nabla n_i d^3 r,$$

where $\phi_e$ is the potential produced by the electrons. We substitute in for $n_i$ the perturbation which arises during supersonic Langmuir collapse.[8] To avoid having to deal with the evolution of the sound after the collapse, we assume that the sound is strongly damped. The function $n_i(\mathbf{r})$ is then a function of even parity, $\int n_i d^3 r = 0$; i.e., from the standpoint of long-wave perturbations the density $n_i$ is similar to the second derivative of a $\delta$ function. We assume that the electron wave is one-dimensional, and we pursue the calculations to an accuracy within a coefficient. The perturbation of the electron density, $n_e = (vt - z)^{-1/2} \theta(vt - z)$, give rise to a field with potential $\phi_e = (vt - z)^{3/2} \theta(vt - z)$. Using $n_i = \delta''(z)$, we find $\ddot{d}(t) = t^{-3/2} \theta(t)$, which in turn gives us

$$W_n \propto n .$$

This expression is valid for $1 \ll n \ll v/(L\omega_0)$, where $L$ is the dimension of the collapsed caviton. At $n \gg v/(L\omega_0)$, the intensity falls off exponentially.

These two examples do not exhaust the possible cases of harmonic emission when there is a finite nonlinearity. Some other examples and also a breaking in the kinetics will be taken up in a more detailed paper. Breaking of an electron wave may occur in situations other than a laser corona, and the resulting emission will play the same role, in the case of a finite nonlinearity, as that played by the coalescence of Langmuir waves into an electromagnetic wave in the case of a slight nonlinearity.


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