Supersonic stabilization of a tangential shear in a thin atmosphere

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A tangential shear in a thin atmosphere is stable if the amplitude of the velocity jump $V > 2\sqrt{2}c_g$, where c_g is the velocity of surface gravity waves. The Coriolis force and the finite width of the flow region are destabilizing factors.

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A tangential shear in an incompressible fluid is absolutely unstable. Taking into account the compressibility of the medium, i.e., the finiteness of the velocity of sound V_s, stabilizes the shear[1] relative to two-dimensional perturbation, if the amplitude of the velocity jump $V > 2\sqrt{2}V_s$. On the other hand, in the case of three-dimensional perturbations, the shear is unstable in a compressible medium as well.[2] Nevertheless, "supersonic" stabilization of the shear is possible, if perturbations with wave vectors **k**, which are not parallel to the flow plane, for some reason cannot occur or are not significant. For example, the proximity of the bottom in the shallow-water approximation gives rise to nearly two-dimensional flow. The role of the velocity of sound in this case is played by the velocity of surface gravity waves $c_g = \sqrt{gH}$, where H is the depth of the liquid, and g is the acceleration of gravity. As will be shown below, stabilization can indeed occur when the condition

$$V > 2\sqrt{2}\, c_g \tag{1}$$

is satisfied. Threshold excitation of nonlinear fluid flows (vortices) was observed in

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experiments described in Ref. 3 by modeling the generation of cyclones and anticyclones accompanying a decrease in the amplitude of the velocity jump in a tangential shear. The physical meaning of the threshold instability was not clear. A detailed experimental check,[3] proposed in this paper, of the mechanism of shear stabilization showed good quantitative agreement between the experimental results and criterion (1). In what follows, we present a derivation of the criterion (1) itself, and we discuss the possible effect of the Coriolis force and finite width of the flow region on the stability of a tangential shear in shallow water.

Large-scale flows in a thin rotating atmosphere are described by the equation[4]

$$\begin{cases} \dfrac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\vec{\nabla})\mathbf{V} = -g\vec{\nabla} H + \Omega\,[\mathbf{V}\times \mathbf{e}_z] \\ \dfrac{\partial H}{\partial t} + \vec{\nabla}(H\mathbf{V}) = 0 . \end{cases} \tag{2}$$

Here $\vec{\nabla} = \mathbf{e}_x\partial/\partial x + \mathbf{e}_y\partial/\partial y$, $\mathbf{e}_x$ and $\mathbf{e}_y$ are oriented toward the east and toward the north; $\mathbf{e}_z = [\mathbf{e}_x\times\mathbf{e}_y]$, and Ω is the Coriolis parameter. Below we shall study the tangential shear

$$\begin{cases} \mathbf{V}_0 = \mathbf{e}_x V_0 \operatorname{sign}(y) \\ g\dfrac{dH_0}{dy} = -\Omega V_0 \operatorname{sign}(y) \end{cases} \tag{3}$$

as the main flow and, for simplicity, we shall set $\Omega = \Omega_0 = \text{const}$. Since under real conditions such flows are restricted in the meridional (along y) direction, we shall also examine the shear in a channel of width $2a$ (i.e., $|y| < a$), allowing, however, the case $ak \to \infty$.

Ignoring the Coriolis force ($\Omega_0 = 0$), the dispersion relation for the phase velocity of the oscillations $c(k)$, as usual,[1] is obtained from the conditions that the depth of the liquid is continuous at the surface of the shear and that this surface is impenetrable:

$$\begin{gathered} (c - V_0)^2\frac{\operatorname{cth}(z_1)}{z_1} + (c + V_0)^2\frac{\operatorname{cth}(z_2)}{z_2} = 0 , \\ z_{1,2} = ak\sqrt{1 - \frac{(c \mp V_0)^2}{gH_0}} . \end{gathered} \tag{4}$$

For the unstable branches of the oscillations, $\mathrm{Im}(c)\neq 0$. For this reason, in an infinitely wide channel ($ak = \infty$), solutions of Eq. (4) corresponding to unstable oscillations have the form

$$c^2 = V_0^2 + gH_0 - \sqrt{(gH_0)^2 + 4gH_0V_0^2} . \tag{5}$$

It follows that instability is possible only for $V_0^2 < 2gH_0$, which is the analog of the supersonic stabilization of the tangential shear examined in Ref. 1. If, on the other hand, the width of the channel is large, but finite, $ak \gg 1$, then the stabilization effect generally disappears, since the increments in this case of the oscillations vanish only for certain values of the flow velocity

$$\frac{V_0^2}{g H_0} = 1 + \left[\frac{\pi(2m+1)}{2ak} \right]^2,$$

where $m = 0, \pm 1, \pm 2....$ Between these values of V_0, the increments γ differ from 0, although they are small

$$|\gamma| \propto |k V_0 \frac{\ln a k}{a k}|. \tag{6}$$

In any case, they are much less than the usual Kelvin-Helmholtz instability increments ($\propto |kV_0|$). Under the experimental conditions, such small γ can be suppressed as a result of the finite (nonzero) width of the region of the velocity jump.

Inclusion of the Coriolis force leads to the appearance of a characteristic dimension of the problem, namely, the Rossby radius $r_0 = \sqrt{gH_0}/\Omega_0$. Another consequence of the condition $\Omega_0 \neq 0$ is that H is not constant [see (3)] and, in addition, the width of the supersonic flow region, where $V_0^2 > gH_0$, cannot exceed V_0/Ω_0. It is clear that the role of the Coriolis force in the stability of the tangential shear is determined by the values of the dimensionless parameters (kr_0) and (V_0/c_g). If $kr_0 \gg 1$, then the effect of the Coriolis force is insignificant and the shear is stabilized according to (5) and (1). On the other hand, for $kr_0 \ll 1$, $V_0/c_g \ll 1$, the flow is geostrophic, $\mathbf{V} \approx (g/\Omega_0)[\mathbf{e}_z \times \nabla H]$, and therefore, incompressible. In this case, according to Ref. 5, the shear is unstable and, in addition, the increments are approximately the same as in the Kelvin-Helmholtz instability. If, finally, $kr_0 \ll 1$, $V_0/c_g \gg 1$, then the increments of the oscillations are small compared to $(V_0/c_g)^{-1}$, and in the limit $V_0/c_g \to \infty$, the flow is stabilized.

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