Stabilization of tangential shear instability in shallow water with "supersonic" fluid flow

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It is demonstrated experimentally that under conditions of (two-dimensional) "shallow water" Landau's conclusion[1] that the instability of the supersonic tangential shear is stabilized for $u > (2g^*H_0)^{1/2}$, where $2u$ is the relative velocity of countermoving flows, H_0 is the depth of the fluid, and g^* is the effective acceleration of gravity, is correct. The result obtained corresponds to the theory of S. V. Bazdenkov and O. P. Pogutse [Pis'ma Zh. Eksp. Teor. Fiz. **37**, 317 (1983)].

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In 1944, Landau[1] came to the conclusion that a tangential velocity shear in a two-dimensional unbounded flow of a homogeneous compressible fluid must be stable if the magnitude of the velocity jump $2u$ satisfies the condition

$$u > \sqrt{2}\, c_s, \tag{1}$$

where c_s is the velocity of sound. Later, Syrovatskiĭ[2] showed that this conclusion is incorrect in general, since it is not based on an analysis of three-dimensional perturbations, but only two-dimensional perturbations: In Ref. 1, perturbations along the flow velocity (X axis) and perpendicular to the plane of the shear (Y axis) were included, but perturbations along the Z axis, perpendicular to the X and Y axes, were not included; in other words, the wave number k_z of the perturbations along the Z axis was assumed to be 0. It was shown in Ref. 2 that a tangential shear is not stable under all conditions, in particular, condition (1), with respect to three-dimensional perturbations ($k_z \neq 0$).

Recently, Bazdenkov and Pogutse[3] re-examined the instability of a tangential shear, but with a different geometry, namely, under conditions of shallow water, whose depth H_0, i.e., the dimension along the Z axis, is negligibly small compared to the scales λ of perturbations along the X and Y axes. It was shown theoretically in Ref. 3 that the tangential shear in shallow water must be stable if the condition

$$u > (2gH_0)^{1/2}, \tag{2}$$

is satisfied, where g is the acceleration of gravity. It is easy to see that (2) and (1) are equivalent: In the case of shallow water, the characteristic velocity of gravity waves $(gH_0)^{1/2}$ plays the role of the velocity of sound. Stability exists in such a geometry because for $H_0 \ll \lambda$ perturbations along the Z axis can indeed be ignored ($k_z \to 0$), and a model,[1] to which the criticisms in Ref. 2 no longer apply, is realized.

In our work, we addressed the problem of verifying experimentally the criterion of supersonic stabilization of a tangential shear instability in shallow water. The experiments were performed with the same setup used to study the Kelvin-Helmholtz

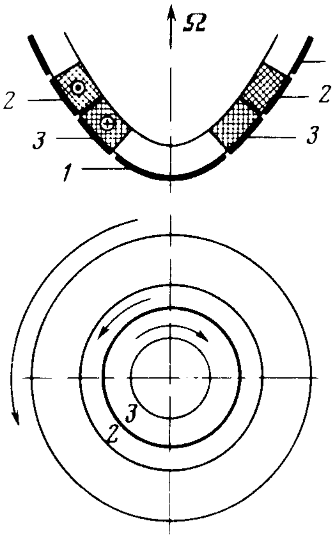

FIG. 1. Experimental setup (see Ref. 4). 1—Paraboloid of revolution with diameter 28 cm (along the upper edge); 2 and 3—rings rotating in opposite directions relative to the paraboloid. Layers of water supported by the rings are shaded. The location of the tangential shear is indicated schematically by the thick lines between the rings. The large arrow in the upper part of the figure indicates the direction of global rotation of the paraboloid and the small arrows indicate the direction of rotation of the rings.

instability of countermoving flows in shallow water in a rotating system.[4] The basic setup (shown in Fig. 1) consisted of a vessel with a parabolic bottom rotating around the vertical axis with angular velocity $\Omega_0 \simeq 11\ s^{-1}$, so that the water was situated along the surface of the vessel in a uniform layer with approximately constant depth. Two independent rings, each 4.5 cm wide, separated by a distance of 1 mm, were placed at the bottom of the vessel. The rings could rotate around the vertical symmetry axis in opposite directions, so that the angular velocities of the rings relative to the paraboloid had identical absolute magnitudes. The frequency of rotation of the rings relative to the paraboloid was varied, but always remained less than the frequency of the global rotation of the vessel. The rings dragged along fluid layers situated above them in local rotation and, in this manner, created countermoving flows in the rotating system of coordinates. The velocity of these flows on the water surface, as measurements showed, was approximately 1.5 times smaller than the velocities of the rings. At the boundary between the rings, there was a discontinuity in the velocity of the flows, whose characteristic width Δ was approximately equal to the depth of the fluid; the latter was varied in the range 5–20 mm. The experiments showed that if the velocity of the countermoving flows u exceeds some threshold u_1, then an instability arises in the system, leading to formation of vortices, whose dimensions λ along the surface of the water always greatly exceed the depth of the fluid H_0 and the width of the discontinuity Δ. The number of vortices fitting into the perimeter of the line of discontinuity depends on the subcriticality $\delta u = u - u_1$: For small δu, eight vortices are observed along a discontinuity of length 63 cm (the size of each vortex is about 8 cm); as the velocity of the countermoving flows increases, the number of vortices on the line of

FIG. 2. Excitation of the fourth mode of the Kelvin-Helmholtz instability. The line of discontinuity in the flow velocity passes through the center of the vortices.

discontinuity (the perturbation mode number) decreases, while the sizes of the vortices increase correspondingly; the minimum number of vortices is 3. A typical flow instability is shown in Fig. 2, and the dependence of the size of the vortices λ on the velocity of the countermoving flows is shown in Fig. 3. The relative velocity of the flows is $2u$, and the vorticity of the flows is parallel to the vector $\vec{\Omega}_0$.

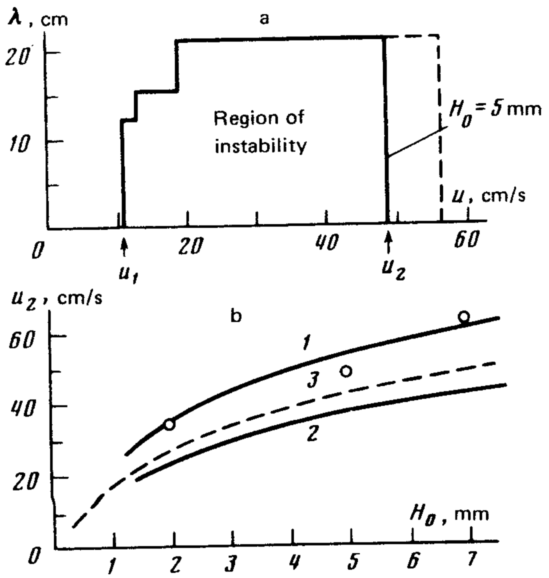

FIG. 3. a—Azimuthal size of vortices as a function of flow velocity, measured at the location of the shear, on the water surface; b—threshold u_2 of stabilization of the tangential shear instability as a function of the fluid depth: 1—velocity of rings; 2—flow velocity; 3—theoretical velocity 3a.

The most interesting fact, which is illustrated in Fig. 3 and is the subject of this investigation, is that the instability being examined does not occur for sufficiently high velocity of countermoving flows, exceeding the threshold u_2. In this case, the flows are laminar and the trajectories of test particles on the surface of the fluid look like concentric circles with centers on the axis of symmetry of the system. The transition through the second stability threshold occurs with a jump, without intermediate states characteristic of the transition through the first threshold (Fig. 3a). Correspondingly, when the flow velocity decreases from quite high magnitudes to magnitudes less than the threshold u_2, three large vortices are formed. As the velocity decreases further, four vortices are formed (Fig. 2), followed by a larger number of vortices. When the threshold u_1 is crossed, the system returns to a stable state with laminar flows. Figure 3b shows the dependence of the velocity u_2 of the second stability threshold (as the threshold is approached from the high velocity side)[1)] on the depth of the fluid H_0 at the location of the discontinuity in the flow velocity. The three curves in Fig. 3b are interpreted as follows: The upper curve is the velocity of the rings relative to the paraboloid and the lower curve is the flow velocity measured on the surface of the fluid; the dashed curve is the velocity equal to $(2g^*H_0)^{1/2}$, where $g^* = g/\cos\alpha$ is the resultant force formed by gravity and the centrifugal force from the global rotation of the liquid (see Ref. 5) and α is the angle between the angular velocity vector of the rotation of the system as a whole and the normal to the surface of the paraboloid at the location of the discontinuity in the flow velocity (in our experiments $\cos\alpha \simeq 0.6$). Thus, it is evident from Fig. 3b that the instability of the tangential shear is indeed stabilized with supersonic fluid flow if

$$u > (2g^*H_0)^{1/2} , \tag{3}$$

consistent with condition (2), predicted by the theory in Ref. 3.

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[1)]The instability under study exhibits hysteresis. In particular, the threshold u_2 increases as it is approached from the low velocity side (Fig. 3a).

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