Light scattering by free holes in semiconductors with a complex valence band

V. A. Voĭtenko, I. P. Ipatova, and A. V. Subashiev
A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted 25 January 1983; resubmitted 1 March 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 7, 334–337 (5 April 1983)

A new unscreened mechanism for light scattering by free holes has been found in semiconductors with a valence band of symmetry Γ_8. This scattering results from fluctuations in the density of the quadrupole moment of the holes. Although heavy holes do the scattering, the mass of the light holes determines the Thomson cross section.

PACS numbers: 78.20.Jq

At a high current-carrier concentration n the conventional mechanism for light scattering by free carriers involves "fluctuations of the effective mass" caused by an anisotropy of the energy spectrum.[1–3] In the present letter we show that in the case of degenerate bands there is another scattering mechanism, which involves fluctuations in the density of the quadrupole moment of the holes. In semiconductors whose hole constant-energy surfaces have a large ripple (Si, GaAs), this new mechanism changes the nature of the scattering, while in semiconductors with a slight ripple and a large difference between the effective masses of the light holes (m_L) and the heavy holes (m_H) (Ge) this mechanism is the sole mechanism for scattering with a small change in frequency at low temperatures T.

Bir and Pikus[4] have shown that the Hamiltonian for the interaction of holes with an energy $\epsilon \ll \Delta$ (Δ is the spin-orbit energy) with an electromagnetic field of frequency $\omega_I \ll E_g/\hbar$, where E_g is the width of the energy gap of the semiconductor, can be found from the Lattinger Hamiltonian, by replacing the ordinary momentum by the generalized momentum and by singling out the terms containing the field. Since the light- and heavy-hole states have identical parity, as in the case studied in Refs. 1 and 2, the scattering is dominated by a term quadratic in the field which can be written conveniently,

$$\mathcal{H}_{int} = \frac{e^2}{2mc^2}(\gamma_1 A^2 - \gamma A_i Q_{ik} A_k), \tag{1}$$

where **A** is the vector potential of the electromagnetic radiation, the operator

$$\hat{Q}_{ik} = \hat{J}_i\hat{J}_k + \hat{J}_k\hat{J}_i - \frac{2}{3}\hat{\mathbf{J}}^2\delta_{ik} \tag{2}$$

represents the dimensionless quadrupole moment of the holes, the operator $\hat{\mathbf{J}}$ represents the total angular momentum of a hole, γ and γ_1 are parameters of the Lattinger Hamiltonian, and m is the mass of a free electron. The first term in (1) gives rise to a scalar scattering by fluctuations of the hole density, which are screened. We will therefore ignore this term. The differential cross section for scattering of a photon $(\mathbf{k}_I,\mathbf{e}_I,\omega_I)$ into the state $(\mathbf{k}_S,\mathbf{e}_S,\omega_S)$ is

$$\frac{d^2\sigma}{d\omega\, d\Omega} = \frac{1}{\pi}\left(\frac{e^2\gamma}{mc^2}\right)^2 [1-\exp(-\hbar\omega/T)]^{-1} e_i^{*I} e_l^S e_k^I e_n^{*S}$$

$$\times \operatorname{Re}\int_0^\infty dt e^{i\omega t} \langle [Q_{il}(\mathbf{q},t), Q_{kn}(-\mathbf{q},0)]\rangle, \tag{3}$$

where $\mathbf{q} = \mathbf{k}_I - \mathbf{k}_S, \omega = \omega_I - \omega_S$, and $Q_{ik}(q,t)$ is a Fourier component of operator (2), written in the second-quantization representation.

Under the condition $m_H \gg m_L$, and at low frequencies $\omega \ll \zeta/\hbar$ (ζ is the chemical potential of the holes), only intrasubband light scattering is important. Since the heavy-hole state density is higher than the light-hole state density, the light is scattered by the heavy holes.

In calculating the ω dependence of $d^2\sigma/d\omega d\Omega$ we focus on the case of infrequent collisions, $qv_F\tau \gg 1$, where v_F is the Fermi velocity of the heavy holes, and τ is their relaxation time. From energy and momentum conservation during the scattering,

$$\epsilon(\mathbf{p} + \hbar\mathbf{q}) - \epsilon(\mathbf{p}) = \hbar\omega \tag{4}$$

we find that $d^2\sigma/d\omega d\Omega$ is nonzero only at $\omega \leqslant qv_F$. In the case of interest, therefore, we must consider the spatial dispersion.[5] To calculate the retarding correlation function in (3) we therefore use the kinetic equation incorporating the electric field $\mathbf{E}(\mathbf{q},\omega)$ produced by the charged particles themselves.[5] A kinetic equation of this type has been solved in the collisionless limit by Gurevich *et al.*[6] in a study of the absorption of longitudinal sound. Corresponding calculations yield

$$\frac{d^2\sigma}{d\omega d\Omega} = \left(\frac{e^2\gamma}{mc^2}\right)^2 \frac{\hbar\omega}{1-\exp(-\hbar\omega/T)} \int \frac{2Vd^3p}{(2\pi\hbar)^3}\frac{\partial f_0}{\partial \xi}\left|Q(\mathbf{p}) + i\frac{eE(\mathbf{q}\omega)}{q}\right|^2 \delta(\omega - \mathbf{q}\mathbf{v}), \tag{5}$$

where V is the crystal volume, $f_0(\epsilon)$ is the Fermi distribution function, and $Q(\mathbf{p})$ is the convolution of operator (2), averaged over the heavy-hole state with momentum $\mathbf{p}$, with the vectors $\mathbf{e}_I$ and $\mathbf{e}_S^*$:

$$Q(\mathbf{p}) \equiv \langle \Psi_T(\mathbf{p}) | e_i^I \hat{Q}_{ik} e_k^S | \Psi_T(\mathbf{p}) \rangle = 3\frac{(\mathbf{p}\mathbf{e}_I)(\mathbf{p}\mathbf{e}_S^*)}{\mathbf{p}^2} - \mathbf{e}_I\mathbf{e}_S^* \cdot \tag{6}$$

With an accuracy to $qr \ll 1$, where r is the screening radius, the electric field is

$$\mathbf{E}(\mathbf{q},\omega) = i\frac{\mathbf{q}}{e}\int d^3p \frac{\mathbf{q}v Q(\mathbf{p})\,\partial f_0/\partial\zeta}{\omega - \mathbf{q}\mathbf{v} + i/\tau}\left[\int d^3p_1 \frac{\mathbf{q}\mathbf{v}_1\,\partial f_0/\partial\zeta}{\omega - \mathbf{q}\mathbf{v}_1 + i/\tau}\right]^{-1}. \tag{7}$$

It can be seen from (5) that in the collisionless limit the parts of $d^2\sigma/d\omega d\Omega$ due to fluctuations of $\mathbf{E}(\mathbf{q},\omega)$ and of the quadrupole momentum $Q(\mathbf{p})$ can be distinguished easily; the $\mathbf{p}$-independent term of $Q(\mathbf{p})$ makes essentially no contribution to $d^2\sigma/d\omega d\Omega$ because of the screening. In particular, these contributions to the cross section are simply additive in the backscattering of linearly polarized light. In this case, under the condition $n \gg (m_H T)^{3/2}/\hbar^3$, the cross section can be written

$$\frac{d^2\sigma}{d\omega\, d\Omega} = V\left[F_1(\mathbf{q},\omega) + (\mathbf{e}_I\mathbf{e}_S)^2 F_2(\mathbf{q},\omega)\right], \tag{8}$$

where

$$F_1(\mathbf{q},\omega) = \left(\frac{e^2\gamma}{mc^2}\right)^2 \frac{\hbar\omega/qv_F}{1-\exp(-\hbar\omega/T)}\frac{27n}{32\epsilon_F}\theta\left(1-\frac{\omega}{qv_F}\right)\left[1-\left(\frac{\omega}{qv_F}\right)^2\right]^2 \tag{9}$$

is the term due to fluctuations in the hole quadrupole moment, while

$$F_2(\mathbf{q},\omega) = \left(\frac{e^2\gamma}{mc^2}\right)^2 \frac{\hbar\omega/qv_F}{1-\exp(-\hbar\omega/T)}\frac{3n}{4\epsilon_F}\theta\left(1-\frac{\omega}{qv_F}\right)\left[\left(2-\frac{\omega}{qv_F}\ln\frac{1+\omega/qv_F}{1-\omega/qv_F}\right)^2 + \pi^2\left(\frac{\omega}{qv_F}\right)^2\right]^{-1} \tag{10}$$

is the term due to fluctuations of the field $\mathbf{E}(\mathbf{q},\omega)$. The two contributions are shown in Fig. 1. We see from expression (8) that in the case $\mathbf{e}_I \perp \mathbf{e}_S$ the cross section is determined by $F_1(\mathbf{q},\omega)$. In the case $\mathbf{e}_I \| \mathbf{e}_S$, both $F_1(\mathbf{q},\omega)$ and $F_2(\mathbf{q},\omega)$ contribute to the cross section; near $\omega = qv_F$ we have $F_2 \gg F_1$. The reason why the fluctuations of the density of the hole quadrupole moment are relatively ineffective near $\omega = qv_F$ is that, as follows from (4), the only holes which can become involved in backscattering in this frequency range are those whose momenta are parallel to $\mathbf{q}$. It can be seen from (6), however, that the unscreened part of the quadrupole moment of these holes is transverse with respect to the vectors $\mathbf{e}_I$ and $\mathbf{e}_S$ in the case of backscattering. The part of $d^2\sigma/d\omega d\Omega$ due to fluctuations of $\mathbf{E}(\mathbf{q},\omega)$ is determined by the imaginary part of the reciprocal longitudinal dielectric function of the holes, which has a threshold singularity at $\omega \sim qv_F$: $F_2(\mathbf{q},\omega) \propto \ln^{-2} 2\omega/(\omega - qv_F)$.

At high temperatures, $T \gg \hbar(\omega_I - \omega_S)$, the fluctuations are classical.[7] In this case the integral scattering cross section $d\sigma/d\Omega$ can be calculated without making any

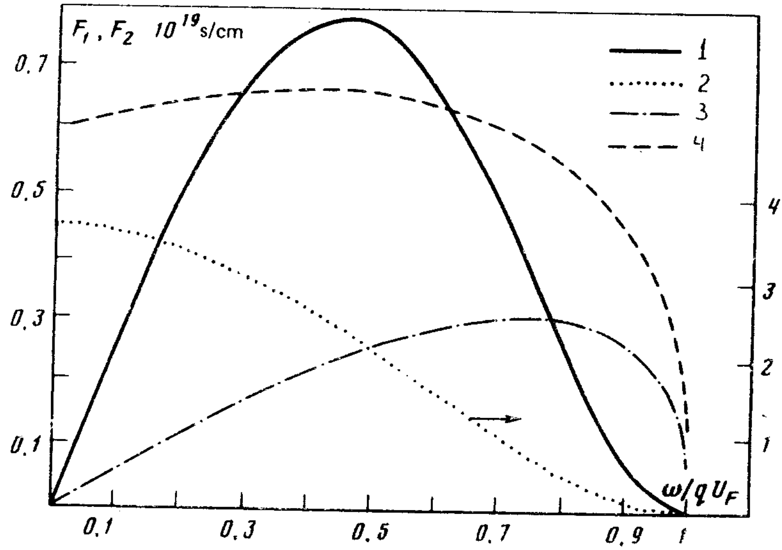

FIG. 1. 1, 2—The function $F_1(\mathbf{q},\omega)$; 3, 4—$F_2(\mathbf{q},\omega)$ for an isotropic [illegible] the p-Ge valence band with $n = 1.5\times 10^{19}$ cm^{-3}. 1, 3—$T = 2$ K; 2, 4—$T = 300$ K.

assumptions regarding the role played by collisions. It can be seen from (3) that $d\sigma/d\Omega$ is determined by the correlation between the fluctuations of the quadrupole-moment density at the same time. In this case we can set $\mathbf{q} = 0$ in $d\sigma/d\Omega$ (Ref. 8). Incorporating the screening in (3) then leads to the condition that the number of holes is constant. It is clear from symmetry considerations, however, that homogeneous fluctuations of the quadrupole moment and fluctuations of the number of holes are statistically independent. In the statistical averaging we can therefore use, equally successfully, either a canonical distribution or a Gibbs grand canonical distribution.[7] The latter approach is simpler. Integrating (3) over the frequency, and using (6), we take the statistical average, finding

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2\gamma}{mc^2}\right)^2 \frac{VT}{10}\left(3 + 3|\mathbf{e}_I\mathbf{e}_S|^2 - 2|\mathbf{e}_I\mathbf{e}_S^*|^2\right)\frac{\partial n}{\partial \zeta}\,. \tag{11}$$

We see from (11) that the Thomson cross section for the type of light scattering under consideration here is determined by the effective mass $m/\gamma \sim m_L$. For degenerate hole statistics, the number of scattering particles is equal to the number of heavy holes in a layer of thickness of order T near the Fermi surface; for nondegenerate statistics, the number of scattering particles is equal to the total number of heavy holes. The reason for this result is that symmetric intrasubband light scattering results from virtual transitions of holes into a different subband, which are incorporated in Hamiltonian (1).

[1]A. A. Abrikosov and L. A. Fal'kovskiĭ, Zh. Eksp. Teor. Fiz. **40**, 262 (1961).
[2]P. M. Platzman, Phys. Rev. **A 139**, 379 (1965).
[3]M. Chandrasekhar, U. Rossler, and M. Cardona, Phys. Rev. **B 22**, 761 (1980).

[4]G. L. Bir and G. E. Pikus, Simmetriya i deformatsionnye éffekty v poluprovodnikakh (Symmetry and Deformation Effects in Semiconductors), Nauka, Moscow, 1972.
[5]E. M. Lifshitz and L. P. Pitaevskiĭ, Fizicheskaya kinetika (Physical Kinetics), Nauka, Moscow, 1979.
[6]V. L. Gurevich, I. G. Lang, and S. T. Pavlov, Zh. Eksp. Teor. Fiz. **59**, 1679 (1970).
[7]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika, Part 1, Nauka, Moscow, 1976 (Statistical Physics, Pergamon, New York).
[8]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Nauka, Moscow, 1982.

Translated by Dave Parsons
Edited by S. J. Amoretty