Scaling theory of Anderson's transition for interacting electrons

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The presence of spin scattering of conduction electrons distinguishes the metal-insulator transition in a disordered system of interacting electrons from concomitant magnetic transformations and greatly simplifies the problem. A scaling theory is constructed for the case which is most often realized experimentally. It is shown that there exists a single-parameter renormalization group, and the inverse resistance of the specimen (conductance) plays the role of the charge. The frequency and temperature dependences of the conductivity in the critical region are found.

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One of the main unsolved problems in the theory of disordered systems is the role of electron interaction in the metal-insulator transition. A scaling theory was proposed in Ref. 1. In contrast to the scaling theory of the Anderson transition for noninteracting electrons,[2–4] this theory contains two charges: 1) a dimensionless total conductivity $G = \pi^2\hbar/e^2R$ (R is the resistance of the specimen) and 2) a single particle density of states, which, as erroneously assumed in Ref. 1, enters into Einstein's relation relating the conductivity σ and the diffusion coefficient D. Actually,[5–7] $\sigma/e^2D = \partial N/\partial\mu$ (N is the electron density, and μ is the chemical potential of electrons) and it does not change greatly as a result of a metal-insulator transition. In addition, only exchange interaction between electrons leading to corrections to the conductivity that are independent of the interaction constants[8,9] are included in Ref. 1. Renormalizability was proved in Ref. 7 in the presence of electron interaction in perturbation theory, for the case in which all effects in the Cooper channel are missing. It was proposed in Ref. 10 that the interaction in the diffusion channel be classified according to the total spin of the electron and hole j. Here all corrections due to interaction with $j = 0$ are independent of the interaction constants and have a universal character.

If the system contains a paramagnetic impurity, then scattering by these conduction electrons suppresses both Cooper corrections as well as diffusion corrections with $j = 1$.[10,11,12] For this reason, if $T\tau_s \ll \hbar$ and $L \gg \sqrt{D\tau_s}$ (L is the size of the specimen and T is the temperature), then the total correction to the classical conductivity G_0 is determined by the interaction with $j = 0$. For $L < L_T = \sqrt{D/\hbar T}$ and $d = 2$, where d is the dimensionality of the specimen, this contribution has the form

$$G - G_0 = -\ln\frac{L}{l}\ , \tag{1}$$

where l is the free path of electrons. This means that the renormalization group has a single charge G:

$$\frac{\partial \ln G}{\partial \ln L} = \beta(G) \qquad (2)$$

and for $G \to \infty$ $\beta = (d-2) - 1/G$. For small G, the conductivity decreases exponentially with increasing L due to localization of electrons and $\beta = \ln(G/G_0) < 0$. Equation (2) with this asymptotic behavior of the function $\beta(G)$ means that in the presence of electron interaction only the dielectric phase is realized in the one- and two-dimensional cases. In addition, the localization radius ξ is the same as for noninteracting electrons, but without spin scattering. For example, for $d = 2$,

$$\xi \sim l \exp G_0 = l \exp(p_F l/\hbar),$$

where p_F is the Fermi momentum of electrons. We note that in the presence of spin-spin scattering for noninteracting electrons $\xi \sim l \exp(p_F l/\hbar)^2$,[13] i.e., the interaction leads to a sharp decrease in the localization radius with $p_F l/\hbar > 1$.

In the three-dimensional case, there exists an unstable fixed point, corresponding to the mobility threshold and absence of a jump in activity accompanying the metal-insulator transition. However, there are no reasons to assume that the critical index of the correlation radius coincides with the corresponding index in the theory of noninteracting electrons.

As the mobility threshold is approached semiconductors exhibit a large number of singly occupied localized states because of Hubbard repulsion and nonuniform distribution of impurities. Elastic scattering by these states leads to a finite spin relaxation time.[14] For this reason, the theory constructed must describe the metal-insulator transition in semiconductors.

At finite temperatures, according to the scaling hypothesis, the conductivity has the form ($d = 3$)

$$\sigma = \frac{e^2}{\hbar \xi} f(\xi/L_T). \qquad (3)$$

As the transition to the insulating phase is approached, we have $\xi \to \infty$. If $\xi \ll L_T$, then $f(\xi/L_T) = A + B(\xi/L_T)$, where A and B are numbers of the order of 1. In this region the correction to the conductivity is therefore proportional to $\sqrt{T}$.[1,8] For $\xi \gg L_T$

$$\sigma = C \frac{e^2}{\hbar} \sqrt{T/D\hbar} \qquad (C \sim 1). \qquad (4)$$

Using Einstein's relation, we obtain

$$\sigma = \frac{e^2}{\hbar}\left(C^2 \frac{\partial N}{\partial \mu} T\right)^{1/3}; \quad D = \frac{T^{1/3}}{\hbar}\left(C \frac{\partial \mu}{\partial N}\right)^{2/3}; \quad L_T = \left(\frac{C}{T}\frac{\partial \mu}{\partial N}\right)^{1/3}. \qquad (5)$$

We shall now estimate the electron–electron collision time. According to Ref. 15,

$$\frac{\hbar}{\tau_{ee}} \sim \frac{\partial \mu}{\partial N} \frac{1}{L_T^3}. \qquad (6)$$

Substituting (5) into (6), we obtain for $\xi \gg L_T$,

$$\hbar \tau_{ee}^{-1} = aT. \tag{7}$$

Relation (7) shows that in the critical region the condition $T\tau_{ee} \gg \hbar$, i.e., the Fermi liquid description of electrons, breaks down and the lengths L_T and $L_{\text{in}} = \sqrt{D\tau_{ee}}$ assume the same order of magnitude. *In the critical region there are no temperature-dependent scales other than* L_T.

If the frequency of the external field $\Omega \gg T/\hbar$, then the characteristic length $L_\Omega = \sqrt{D/\Omega}$ and for $L_\Omega \ll \xi$

$$\sigma(\Omega) \sim \frac{e^2}{\hbar}\left(\Omega \frac{\partial N}{\partial \mu}\right)^{1/3}. \tag{8}$$

Relation (8) was obtained previously for noninteracting electrons in Refs. 4 and 16–18.

We shall now examine the case in which there are no paramagnetic impurities, but there is strong spin-orbit scattering. This situation is realized, for example, in cubic p-type semiconductors and in heavy metals. Without the electron–electron interaction, for $d = 2$, $\beta(G)$ has the form shown by the dashed line in Fig. 1. For $G \to \infty$ $\beta(G) = G/2\lambda > 0$.

Spin-orbit scattering suppresses the contribution of the interaction with $j = 1$, leaving the contribution of the interaction with $j = 0$ unchanged.[12,10] For this reason, if interaction effects in the Cooper channel are ignored, then for $\sqrt{D\tau_{\text{so}}} = L_{\text{so}} \ll L \ll L_T \lesssim L_{\text{in}}$ (τ_{so} is the spin relaxation time with spin-orbit scattering)

$$G = G_0 - \frac{1}{2}\ln\frac{L}{l}, \tag{9}$$

i.e., for $G \gg 1$ $\beta(G) = d - 2 - 1/2G$. This means that the graph of the function $\beta(G)$ has the form shown by the solid line in Fig. 1 and corresponds to localization of electrons with $d = 2$ and $p_F l \gg \hbar$. In this case, $\xi \sim \xi_0^2/l \gg \xi_0$, where ξ_0 is the localization length of noninteracting electrons with potential scattering. The temperature and frequency dependences of the conductivity with $d = 3$ in the critical region in this case are also described by relations (5) and (8).

In conclusion, we note that the simplification of the problem of the metal-insula-

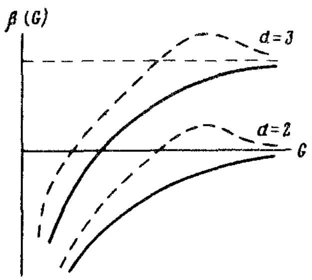

FIG. 1. Gell-Mann–Low function $\beta(G)$ in the presence of spin-orbit scattering in two- and three-dimensional cases. The solid and dashed lines correspond to cases with and without electron-electron interaction.

tor transition, taking into account the finiteness of the spin relaxation time of conduction electrons, is apparently related to the fact that the magnetic transformations accompanying the metal-insulator transition in disordered systems can be ignored in this case.

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